COMPARATIVE ANALYSIS

OF

STEERING ENGINE PROPOSALS

by

ROWLAND GRAYSON EVANS
LIEUTENANT, UNITED STATES NAVY

B.S., U.S. Naval Academy
(1958)

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Submitted in Partial Fulfillment of the
Requirements for the Degree of
Naval Engineer and the Degree of
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at the

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June, 1966

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Department of Naval Architecture and Marine Engineering, 20 May 1966

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Chairman, Departmental Committee on Graduate Students

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Submitted to the Department of Naval Architecture and Marine Engineering in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering and the professional degree, Naval Engineer.

ABSTRACT

This thesis is a survey of the various mechanisms that could possibly prove to be suitable for use as a steering engine for a ship. An initial investigation narrowed the field to the four most promising systems. These were a hydraulic piston operating a rapson slide, a hydraulic rotary vane actuator, a harmonic gear drive, and a ball bearing screw operating a rapson slide. This group was analyzed in detail to minimize weight by optimization of their system parameters and, for the new proposals, to determine feasibility. A control response analysis as well as an investigation of compatibility with the overall ship system was part of the feasibility study.

The results of this analysis are summarized as follows. The hydraulic piston design was optimized with respect to weight by use of the parameter $p/f\sigma_y$. It was determined from this that high hydraulic pressures were desirable, and, as a result of their use, this was found to be the lightest and most efficient system. The rotary vane actuator was found to be attractive because of its simplicity, but it was the heaviest of the proposals. The harmonic gear was found to be the most promising of all the gear reducer drives, and a feasible configuration was developed. However, it was relatively heavy and had the lowest efficiency of the group. Its high compliance reduced the natural frequency of the rudder to the point where vibration of the rudder and hull were possible. The ball bearing screw design was found to be light in weight and reasonably efficient. However, it was found to be unfeasible for the high torque range because adequate lifetime could not be obtained due to manufacturing restrictions which limit the maximum available size.

Thesis Supervisor: Herbert H. Richardson
Title: Associate Professor of Mechanical Engineering

Submitted [illegible]
in partial fulfillment of the [illegible]
Science in [illegible]

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Most of the data for this thesis was not available within the Institute and had to be obtained from the various equipment manufacturers. The author wishes to express his appreciation to the design engineers of these firms that so helpfully supplied all of the requested information. In particular, the author is deeply indebted to Mr. John H. Carlson, Development Engineer, United Shoe Machinery Corporation for all of the time and effort that he so graciously contributed. Without his patient assistance the harmonic gear section would not have been possible.

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CHAPTER I

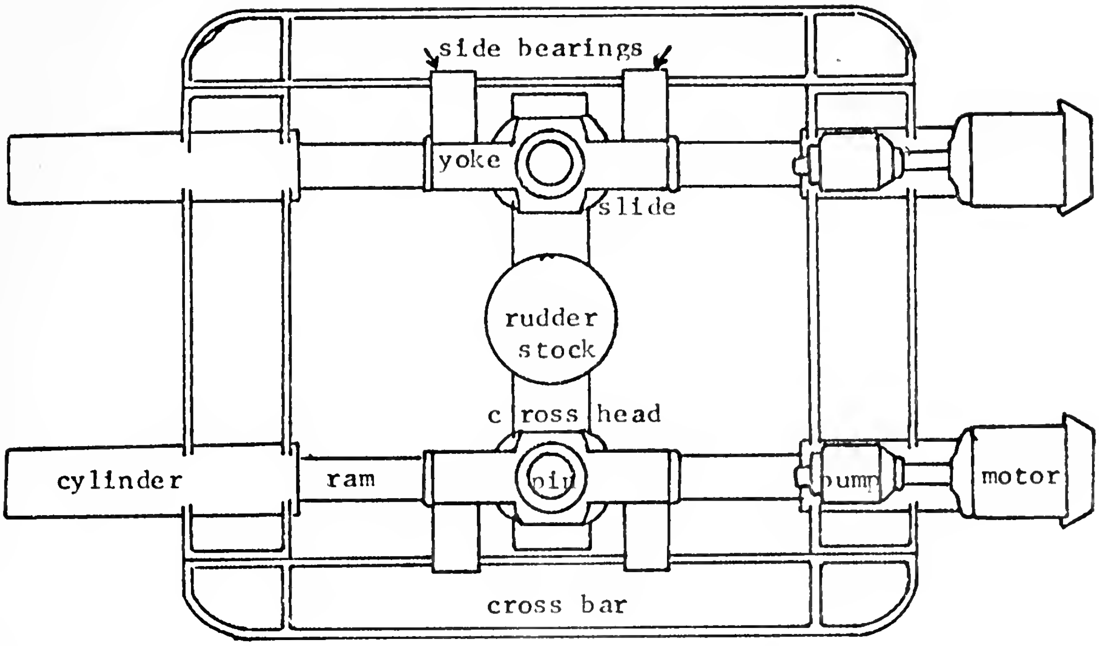
INTRODUCTION

In the recent past we have seen the development of effective automatic steering control equipment and the continuing research into hull response and turning behavior. These are evolutionary steps forward in the solution of the problem of providing the most efficient directional control for ships.

Yet in this same period there has been relatively little change in the steering engine itself. This stagnation has been due mainly to the emergence many years ago of the electro-hydraulic system as markedly superior to all of the other then existing types. The resulting well justified reputation of excellent performance and reliability has generated not only a reluctance to change but also a reluctance to investigate and develop new systems. Certainly the first of these is understandable, but the second cannot be logically defended. It was challenged by Butterfield[1] more than ten years ago; and since then the rotary vane, one of the systems that he recommended, has gained some acceptance, particularly abroad. More recently we have seen the development of a new high pressure rapson slide system in England[2] and a rack and pinion electro-hydraulic system in this country.[3]

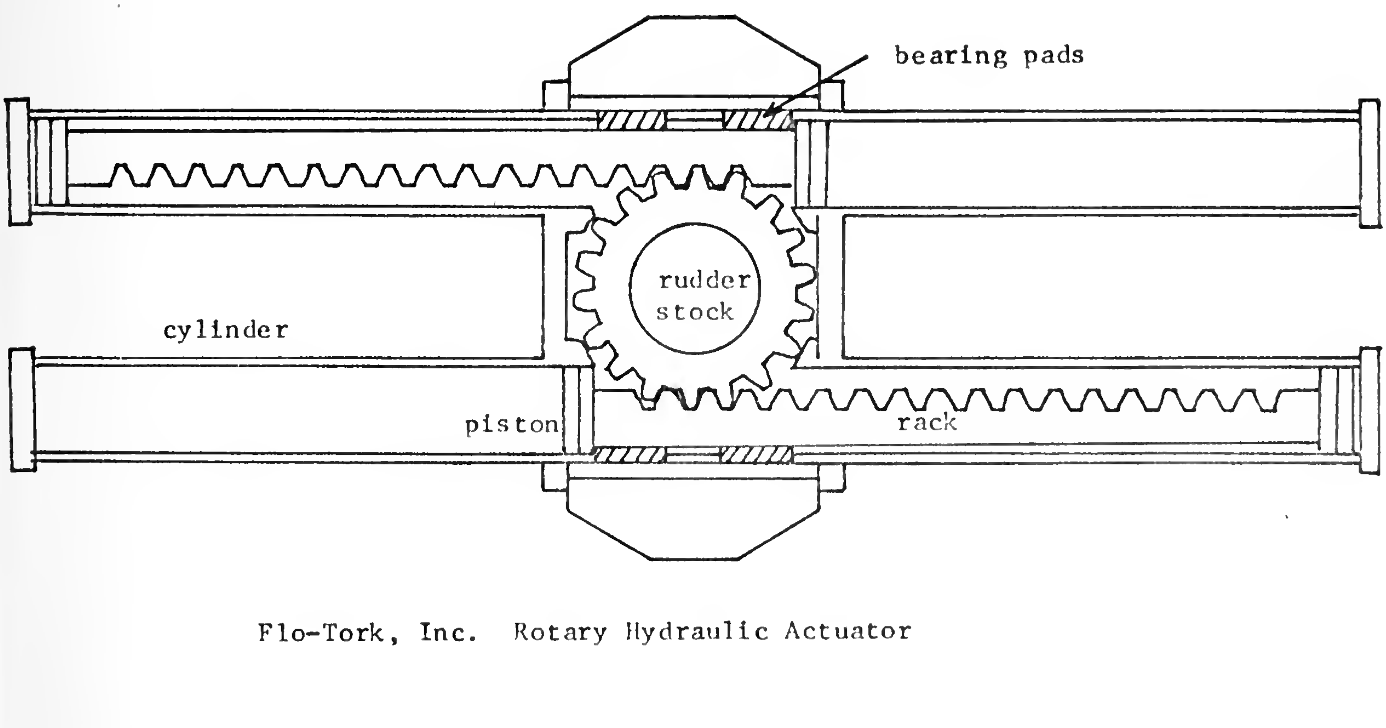
In view of these developments and other recent advances in technology it seems appropriate at this time to conduct a survey of the possible methods of meeting the requirements of a rudder actuator. It is the purpose of this paper to compare several possible proposals as to feasibility, weight and space. Although the cost is certainly a prime consideration in the selection of a steering engine, it is considered to be beyond the scope of the investigation proposed in this paper.

FIGURE I



Brown Bros. and Co., Edinburgh, 5000 psi Ram Type Steering Engine

RECENT STEERING ENGINE DESIGNS



Flo-Tork, Inc. Rotary Hydraulic Actuator

CHAPTER II

PROCEDURE

It is presumed here that an adequate rudder exists and that an appropriate steering control system on the bridge provides rudder angle orders to the steering engine. Other than requiring that the various steering engines proposed be compatible with existing rudders and steering control systems, this paper makes no investigation of those areas.

It became clear very early that a completely analytic investigation of so broad a field was not possible. It was therefore decided that the most profitable approach would be to investigate the moderately high torque range and then attempt to indicate the trend of the results when they are extended into the torque ranges above and below. A rudder stock torque requirement of five million inch pounds was arbitrarily selected.

As the investigation proceeded, it became necessary to estimate the order of magnitude of the characteristics of the rudder and stock with respect to dimensions, stiffness, inertia, and damping. Accordingly the rudder in Fig. I was selected with the torque curves shown in Fig. II. These torques were computed in the standard manner suggested by Taplin[4] including all corrections and using the data of Fehlner and Wicker[5]. The physical inertia and stiffness of the structure was estimated from the dimensions and scantlings and is given in Appendix I. Estimation of the virtual inertia and damping was found to be most easily done by using Appendix I of Paster and Abkowitz's study on torpedo design[6]. It was assumed that the angular velocities used are low enough that the Theordorsen oscillatory correction factor can be taken as 1. These calculations are also included in Appendix I.

It is to be emphasized that this rudder is intended to indicate only in a general way the characteristics that might be found in a five

FIGURE II

REPRESENTATIVE RUDDER

Rudder Area 196.6 ft.^{2}

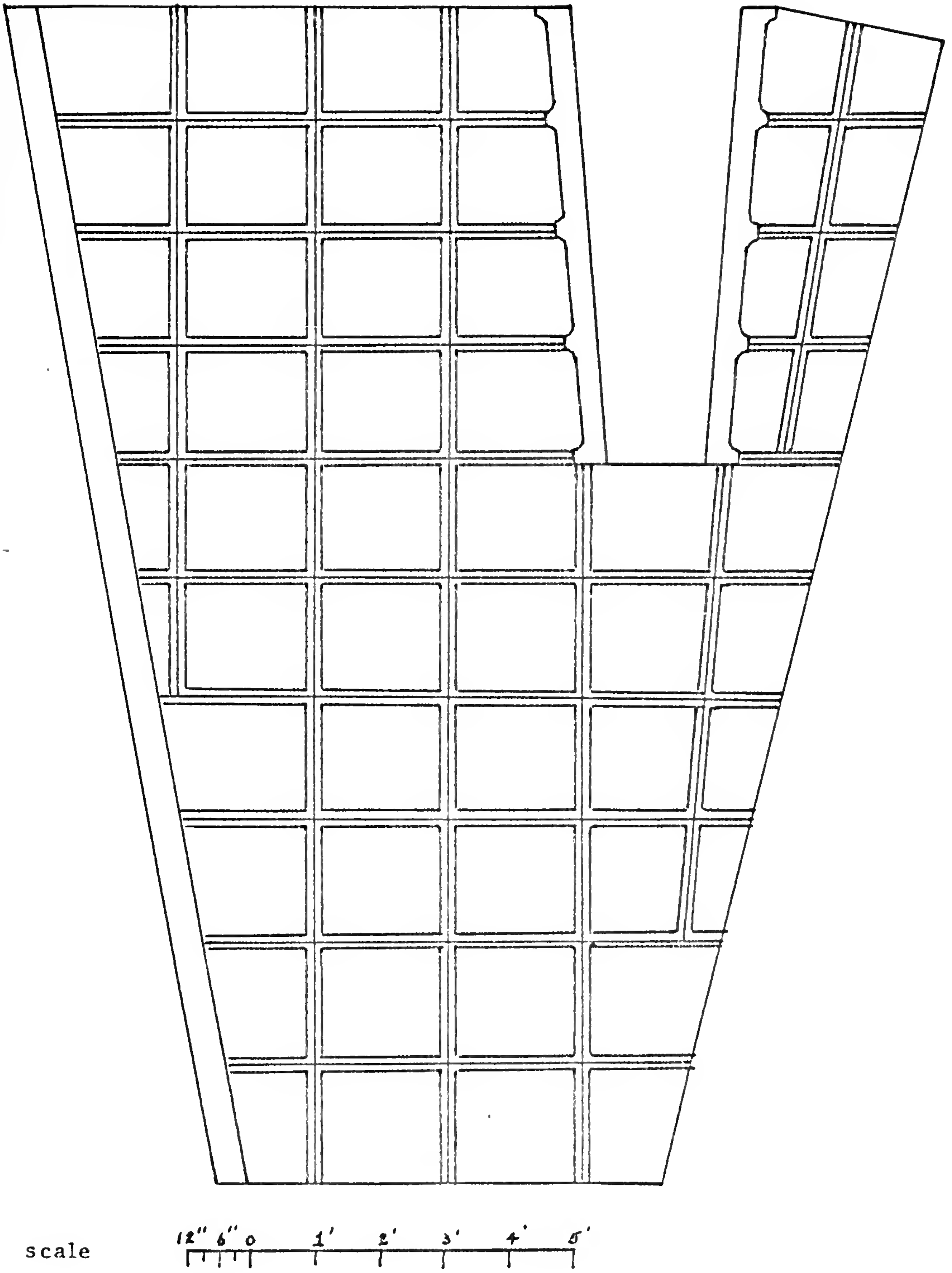
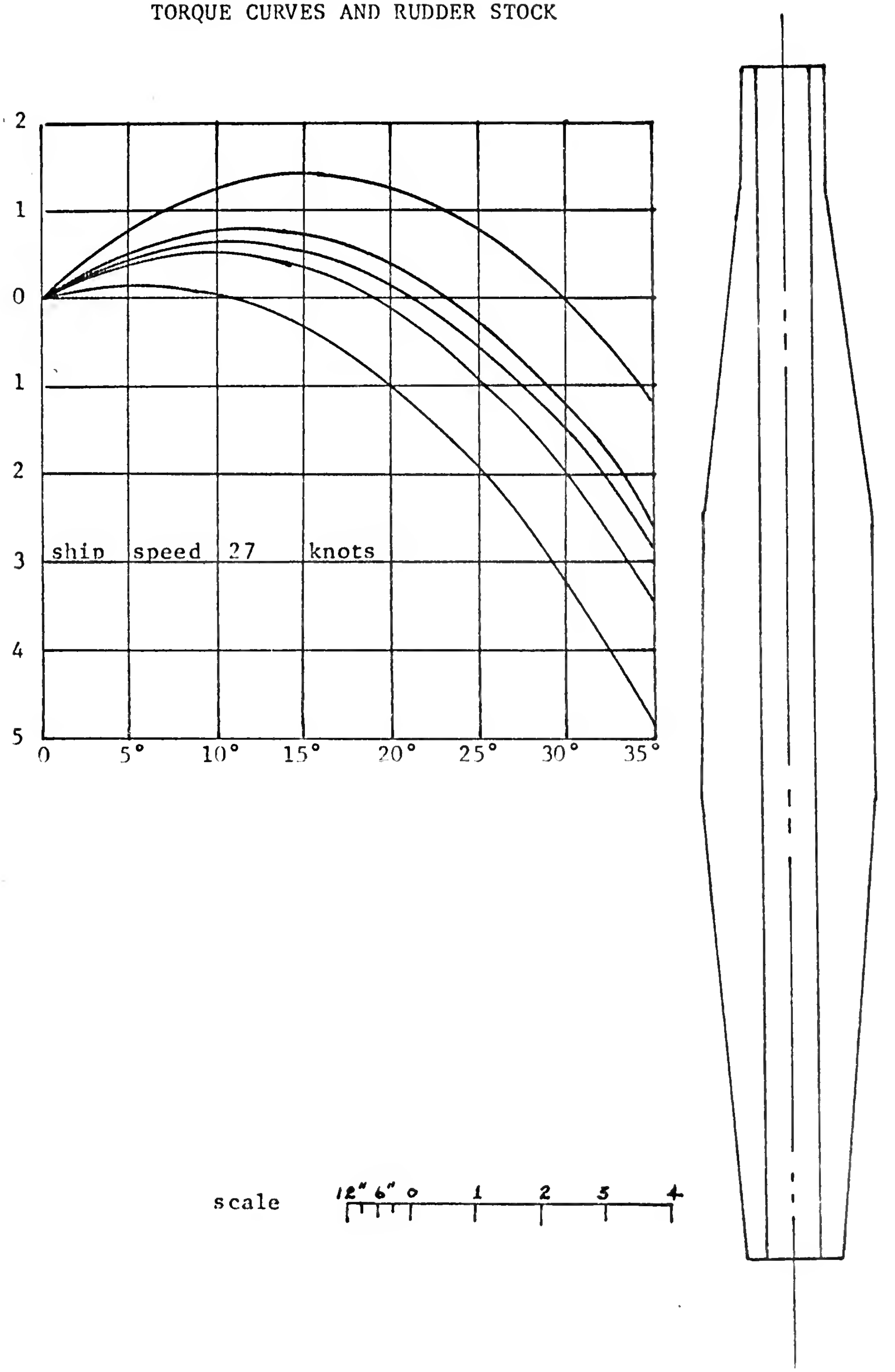


FIGURE III

TORQUE CURVES AND RUDDER STOCK



million inch pound torque rudder. It is recognized that the values obtained could be an order of magnitude different in some other rudder of the same torque. Nevertheless, it is possible to draw some valid general conclusions from this rudder if appropriate consideration is taken of these possible variations.

The required rudder rate is specified to be 2 1/3 °/sec which corresponds to .39 rpm or hard over to hard over of 70° in 30 seconds. This choice is based on the recommendations of Eda and Crane[7] as being the most probable optimum value for most ships. The maximum rudder angle required was arbitrarily selected as 35°. It is recognized that higher rudder rates and maximum angles might be desired. Accordingly it is further required that the proposed systems be adaptable to using these higher values. The required accuracy of the steering engine control system is that the rudder be positioned to within one fourth of a degree of the ordered rudder angle, i.e. static error equals .25°[8].

As the analyses progressed it became necessary to specify the manner in which rudder stock torque varied with rudder angle. Examination of the results of several maneuvering trials reveals that rudder torques and ram pressures are erratic and as yet unpredictable with any accuracy. Furthermore designs vary widely and in many cases maximum torque is reached in the astern condition at considerably less than maximum rudder angles[9]. In the ahead condition the heaviest loadings are often generated in going from hard over to hard over[10]. For the purposes of this study it is entirely adequate to represent rudder torque as a simple linear function of rudder angle. Deviations from this representation can be expected to alter rudder rates. However,

since the power sources used in the proposed systems are essentially constant speed devices under load the variations in rudder rate are expected to be small.

In two of the systems proposed, the problem arose of predicting the expected lifetime of anti-friction bearings. That is the system had to be designed such that the expected lifetime was satisfactory. In these cases, the present Bureau of Ships criteria of a lifetime of 50,000 hours at 51% of maximum load was used. The derivation of this criteria together with a general discussion concerning it are included in Appendix II.

For the purposes of this study it is presumed that the primary source of power to the engine will be electrical and that this will be in the form of 440 volts, three phase, alternating current. Thus if a proposed system requires direct current, the weight and characteristics of the rectification system used must be included in the analysis. This requirement is imposed in order to standardize the analysis and hopefully make it pertinent to a wider variety of ships.

The problem is then defined as one of comparing the designs of an automatic position control device with an arc of travel of 70° and an accuracy of .25° static error which uses 440 volts A.C. electrical power to produce five million inch pounds of torque on a shaft turning at .39 rpm. That is; a high torque, low speed, electrically powered, position servo.

The types of mechanisms which could possibly meet the requirements of the problem are divided into three general categories. In the analysis section of the paper, each of these categories is examined and the most promising systems in each category are selected for further investigation.

The equations for the operation and weight of each system so selected are derived and examined, and some attempt is made at optimizing their parameters. Then a design is worked out and weights and operating characteristics are estimated. In Chapter IV the results of these analyses are compared and discussed. The conclusions of Chapter V are drawn on the basis of these results.

CHAPTER III

ANALYSIS

3.0 GENERAL

The mechanisms which will be investigated may be grouped into three general categories. The first of these is the direct acting electromagnetic devices which can be treated quickly. The next category is that of the electro-hydraulic machines. Two of these will be considered. They are the rotary vane actuator and the linear actuator or piston and cylinder driving through either a rapson slide, a tiller and linkage or a rack and pinion. The final category is the electro-mechanical machines. Although each of these devices may incorporate a hydraulic pump-motor transmission for control purposes, they are distinct from the previous category in that the low speed, high torque is produced principally by mechanical means. Three general types of these will be considered. They are gear reducer drives, the ball bearing screw actuator, and hydrostatic bearing drives.

3.1 DIRECT ACTING ELECTRO-MAGNETIC

The definition of the problem suggests that the first category of devices that should be investigated is direct acting electro-magnetic devices. These could take the physical form of a giant synchromotor, a D.C. motor, or an open squirrel cage motor all attached directly to the rudder stock. In order to get the required torque, enormous windings would have to be employed which, in addition to their great weight and size, would consume large amounts of power in resistive heating. This not only produces low efficiency but a large heat removal problem as well. And, of course, the D.C. devices would have to have their rectifiers.

CHAPTER III

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3.0 GENERAL

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3.1 DIRECT ACTING ELECTRO-MAGNETIC

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Of the three, the most hopeful might be the open squirrel cage shown in Figure IV which requires no rectifier and whose radius might be extended to the point where the required electromagnetic forces were within reason. However, the radius required would be excessively large, and even then the system would be heavier and less efficient then existing ones.

Actually this whole category could have been dismissed immediately by considering the fact that electro-magnetic devices operate most efficiently as low torque high speed machines. This is, of course, the diametric opposite of the requirements of this application. One further implication of this is that if electric motors are used, it will be to drive oil pumps, gear trains, etc. which convert its high speed, low torque to the low speed, high torque required here.

3.2 <u>ELECTRO-HYDRAULIC DEVICES</u>

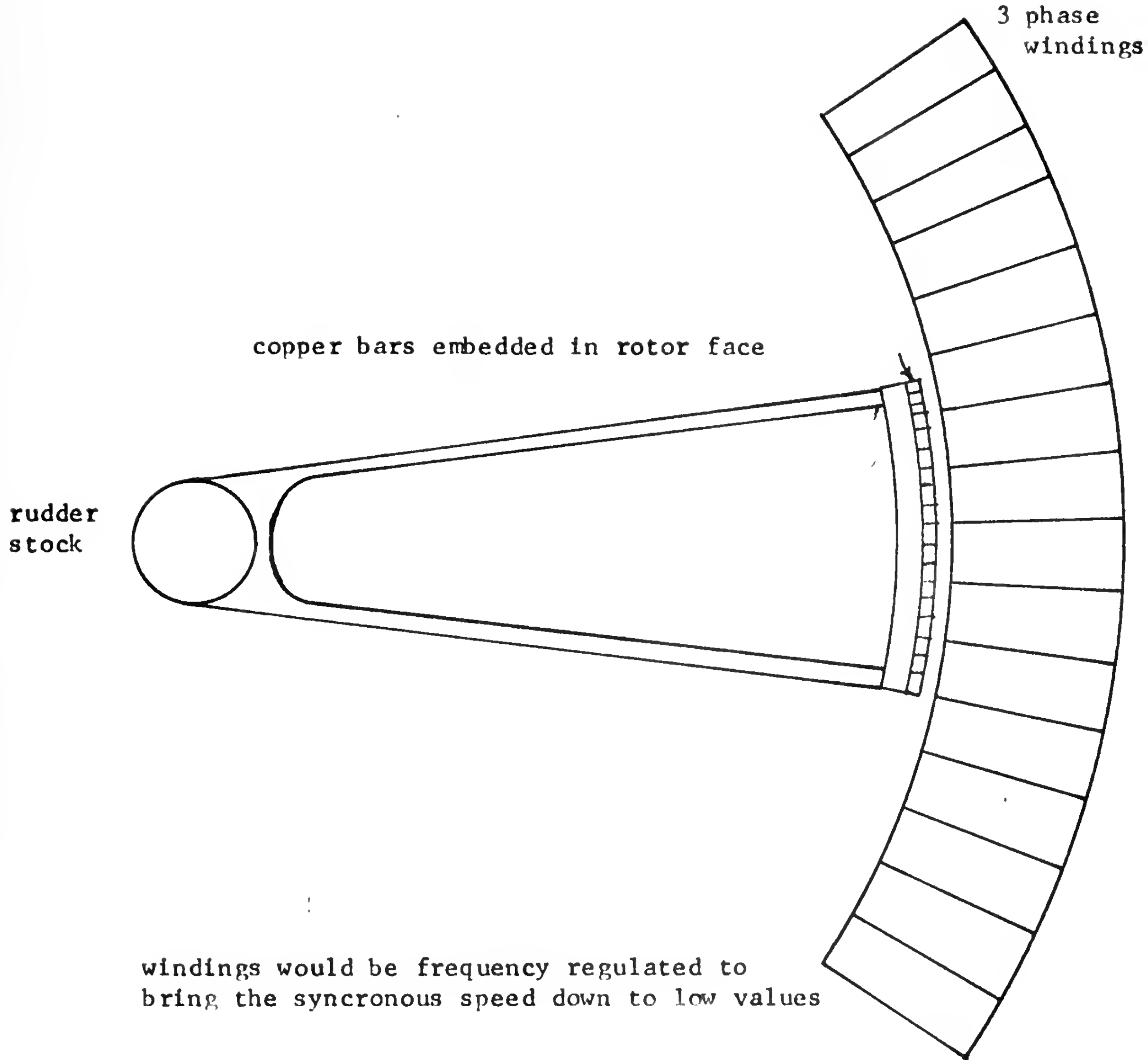
3.2.1 <u>Linear Actuator</u>

3.2.1.0 <u>General</u>

Although the analyses of the various proposed systems in general includes a feasibility study, it is clear that in view of the performance record of the electro-hydraulic piston and cylinder such a study would be redundent. However, a weight and space optimization study is a pertinent area of interest. The general procedure to be used is to attempt to optimize the system with regard to weight, and then examine the effects of this optimization on space. Although limited ship, it is necessary to arbitrarily divide the problem into manageable parts to facilitate the analysis.

FIGURE IV

OPEN SQUIRREL CAGE MOTOR



The linear actuator was divided into 3 components; the piston and cylinder, the pump, and the shaft turning device. The weight optimization procedure will be to first write the equations of operation of the various components of the system. Then the weight equations will be written where possible. These equations will be examined as to the relationships of their parameters. Then the weight optimization problem for the combined system can be defined and a solution attempted.

3.2.1.1 Piston and Cylinder

An appropriate point at which to start the analysis is to consider the piston and cylinder. The best mathematical model to use for calculating stresses in hydraulic cylinders is that of the "thick walled" cylinder. This will apply to the broadest range of cylinders, since at low pressures it will reduce to the case of the "thin walled" cylinder. Further, it is shown in the following analysis that representation by a simple hoop stress calculation does not yield meaningful results.

3.2.1.2 Simple Hoop Stress Equations

$$\sigma = \frac{pd}{2t} = \frac{pr}{t} = f\sigma_y \qquad (1)$$

$$\left(\frac{t}{r}\right) = \frac{p}{f\sigma_y}$$

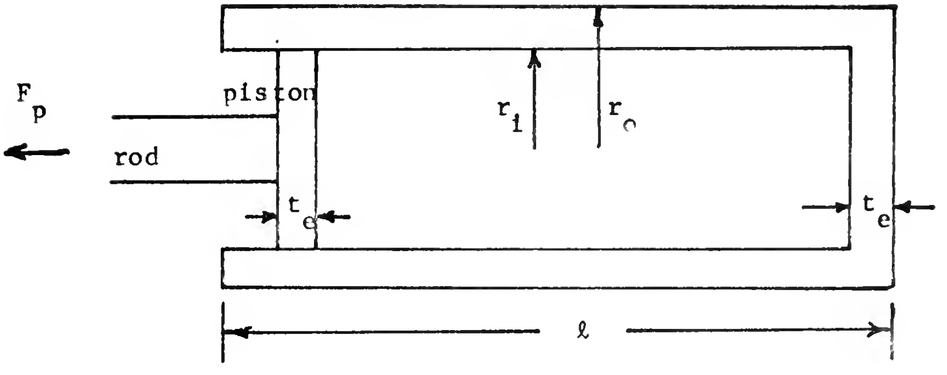
$$W = \gamma_s[2\pi r_1^2 t_e + 2\pi r t \ell] + \gamma_o \pi r_1^2(\ell - 2t_e) = \text{weight} \qquad (2)$$

$$= \pi r_1^2[\gamma_s\{2t_e + 2(\tfrac{t}{r})\ell\} + \gamma_o\{\ell + 2t_e\}]$$

$$= \frac{F_p}{p}[\gamma_s\{2t_e + \frac{2p}{f\sigma_y}\ell\} + \gamma_o\{\ell + 2t_e\}]$$

FIGURE V

PISTON AND CYLINDER NOMENCLATURE



r_i = inner radius

r_o = outer radius

$k = \frac{r_o}{r_i}$

p = pressure (psi)

γ_s = specific weight of steel

γ_o = specific weight of oil

σ_y = yield stress

f = 1/factor of safety

$$W = \frac{F_p}{p}\left[2t_e(\gamma_s+\gamma_o)+\gamma_o\ell\right] + \gamma_s \frac{2F_p\ell}{f\sigma_y} \tag{3}$$

differentiate with respect to pressure

$$W = -\frac{F_p}{p^2}\left[2t_e(\gamma_s+\gamma_o)+\gamma_o\ell\right] + 0 \tag{4}$$

This has no maximum or minimum with p. It can be seen from equation (3) that W goes down monotonically with increasing p due to lower oil weight and end weights. This is not born out by practice so a better model is needed. Next try the thick wall cylinder model.

3.2.1.3 <u>Thick Wall Cylinder Equations</u>[11]

$$\text{Radial stress} \quad \sigma_R = p_i \frac{r_i^2}{r_o^2-r_i^2}\left(1-\frac{r_o^2}{r^2}\right) \tag{5}$$

$$\text{Tangential stress} \quad \sigma_T = p_i \frac{r_i^2}{r_o^2-r_i^2}\left(1+\frac{r_o^2}{r^2}\right) \tag{6}$$

Maximum stress occurs at the inner radius where $r = r_i$

$$\text{Max } \sigma_T = p_i \frac{r_i^2}{r_o^2-r_i^2}\left(1+\frac{r_o^2}{r_i^2}\right) = p\,\frac{r_i^2+r_o^2}{r_o^2-r_i^2} \tag{7}$$

let $r_o = kr_i$

$$\sigma_T = p\,\frac{r_i^2}{r_i^2}\,\frac{(1+k^2)}{(k^2-1)} = p\,\frac{(k^2+1)}{(k^2-1)} \tag{8}$$

$$W = \frac{\pi}{\rho}\left[\rho_a (Y_s + Y_c) + \rho_c (2L)\right] + Y_s \frac{2\pi r_o^3}{\rho} \quad (3)$$

differentiate with respect to pressure

$$W = -\frac{\pi r_o^2}{2\rho^2}\left[2t_s (Y_s + Y_c) r_c x\right] + 0 \quad (4)$$

This has no maximum or minimum with p. It can be seen from equation (4) that W goes down monotonically with increasing p due to lower oil volume and end weights. This is not born out by practice so a better model is needed. Next try the thick wall cylinder model.

3.2.1.3 Thick Wall Cylinder Equation [11]

$$\text{Radial stress } \sigma_R = p_i \frac{r_i^2}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right) \quad (5)$$

$$\text{Tangential stress } \sigma_T = p_i \frac{r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right) \quad (6)$$

Maximum stress occurs at the inner radius where $r = r_i$

$$\text{Max } \sigma_T = p_i \frac{r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) = p \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \quad (7)$$

let $r_o = k r_i$

$$\sigma_T = p \frac{r_i^2}{r_i^2} \frac{(1+k^2)}{(k^2-1)} = p \frac{(k^2+1)}{(k^2-1)} \quad (8)$$

Piston area is related to F_p by the equation

$$F_p = p\pi r_i^2 \tag{9}$$

$$r_i^2 = \frac{F_p}{\pi p} \tag{10}$$

Now consider F_p as a given constant and then try to find the minimum weight for the piston and cylinder. The equation of the weight of cylinder with piston and fluid is:

$$W = \gamma_s[2\pi r_i^2 t_e + \pi(r_o^2 - r_i^2)\ell] + \gamma_o \pi r_i^2(\ell - 2t_e) \tag{11}$$

$$= \pi r_i^2[\gamma_s\{2t_e + (k^2-1)\ell\} + \gamma_o(\ell - 2t_e)] \tag{12}$$

The thickness of the ends t_e can be found from the formula

$$\begin{aligned} \sigma_{max} &= \alpha \frac{pr^2}{t_e^2} \\ &= f\sigma_y \end{aligned} \tag{13}$$

α is a constant which depends on the supports and which would be different for the end and for the piston. From the Catalog of Results, Chapter IV of reference [11], $\alpha = .75$ for clamped edge with pressure loading. It has higher values for centrally supported pressure loaded plates. A reasonable average value for the piston and the fixed end might be $\alpha = 1$.

$$t_e^2 = \frac{\alpha p r^2}{f\sigma_y} \tag{14}$$

$$t_e = r_i\sqrt{\frac{\alpha p}{f\sigma_y}}$$

$$= \sqrt{\frac{F_p}{\pi p}}\ \sqrt{\frac{\alpha p}{f\sigma_y}} = \sqrt{\frac{\alpha F_p}{\pi f\sigma_y}} = \text{function of } f\sigma_y \text{ and } F_p \text{ only} \tag{15}$$

Having calculated the radial and tangential stresses in the cylinder, an appropriate strength theory must be selected to calculate σ_{max}.

The Hencky-Von Mises Shear Energy Theory was chosen because it generally gives the best results for the reasonably ductile steels being considered here.[12] At the inner fiber,

$$\sigma_{max}^2 = \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = f^2\sigma_y^2$$

$$f^2\sigma_y^2 = [p\,\frac{r_i^2+r_o^2}{r_o^2-r_i^2}] + p\,\frac{r_i^2+r_o^2}{r_o^2-r_i^2}\,p + p^2 \tag{16}$$

$$\frac{f^2\sigma_y^2}{p^2} = \left(\frac{k^2+1}{k^2-1}\right)^2 + \left(\frac{k^2+1}{k^2-1}\right) + 1 \tag{17}$$

This equation is in a slightly awkward form, but attempts to reduce it to a function of p only by appropriate simplifications were unsuccessful. The next step then is to solve the quadratic equation for (k^2-1).

$$\frac{f^2\sigma_y^2}{p^2} = \frac{(k^2+1)^2+(k^2+1)(k^2-1)+(k^2-1)^2}{(k^2-1)^2}$$

$$= \frac{k^4+2k^2+1+k^4-1+k^4-2k^2+1}{(k^2-1)^2}$$

$$= \frac{3k^4+1}{(k^2-1)^2} \qquad (18)$$

let $$\frac{f^2\sigma_y^2}{p^2} = C \qquad (19)$$

then $$C(k^4-2k^2+1) = 3k^4+1 \qquad (20)$$

$$k^4(C-3)-2Ck^2+C-1 = 0$$

$$k^2 = \frac{2C}{2(C-3)} \pm \sqrt{\left(\frac{C}{C-3}\right)^2 - \left(\frac{C-1}{C-3}\right)}$$

$$= \frac{C\pm\sqrt{C^2-(C-1)(C-3)}}{C-3} = \frac{C\pm\sqrt{C^2-C^2+4C-3}}{C-3}$$

$$k^2 = \frac{C\pm\sqrt{4C-3}}{C-3} \qquad (21)$$

must use + sign or else k will be less than one which is impossible

$$k^2-1 = \frac{C+\sqrt{4C-3}-C+3}{C-3} = \frac{3+\sqrt{4C-3}}{C-3}$$

$$k^2-1 = \frac{3+\sqrt{\frac{4f^2\sigma_y^2}{p^2}-3}}{\frac{f^2\sigma_y^2}{p^2}-3} = \frac{3p^2+\sqrt{4f^2\sigma_y^2-3p^4}}{f^2\sigma_y^2-3p^2} \qquad (22)$$

Rewriting the weight equation in non-dimensional form

$$\frac{W}{F_p\gamma_s\ell} = \frac{F_p\gamma_s 2t_e}{(F_p\gamma_s\ell)p} + \frac{F_p\gamma_s\ell}{(F_p\gamma_s\ell)}\left(\frac{k^2-1}{p}\right) + \frac{F_p\gamma_o(\ell-2t_e)}{(F_p\gamma_s\ell)p} \tag{23}$$

$$= \frac{1}{p}\left[\frac{2t_e}{\ell} + \frac{\gamma_o}{\gamma_s}\ \frac{(\ell-2t_e)}{\ell}\right] + \frac{k^2-1}{p}$$

$$\frac{Wf\sigma_y}{F_p\gamma_s\ell} = \frac{f\sigma_y}{p}\left[\frac{2t_e}{\ell} + \frac{\gamma_o}{\gamma_s}\ \frac{(\ell-2t_e)}{\ell}\right] + \frac{f\sigma_y}{p}\left[\frac{3+\sqrt{\dfrac{4f^2\sigma_y^2}{p^2} - 3}}{\dfrac{f^2\sigma_y^2}{p^2} - 3}\right] \tag{24}$$

The first term on the right may be rewritten as

$$\frac{f\sigma_y}{p}\left[\frac{2t_e}{\ell}\left(1 - \frac{\gamma_o}{\gamma_s}\right) + \frac{\gamma_o}{\gamma_s}\right] \tag{24a}$$

or as $$\frac{f\sigma_y}{p}\left[2\left(1 - \frac{\gamma_o}{\gamma_s}\right)\sqrt{\frac{\alpha F_p}{\pi f\sigma_y\ell^2}} + \frac{\gamma_o}{\gamma_s}\right] \tag{24b}$$

Plots of this equation for varying values of the non-dimensional end thickness, non-dimensional pressure and non-dimensional weight are given in Figure (VI). The calculations of these curves are given in Appendix III.

Theoretically this weight equation could have been optimized with respect to pressure by setting the partial differential with respect to p equal to zero. This would have yielded an equation for optimum p in terms of the other variables $f\sigma_y$, t_e and ℓ. However, this would have given no information about the shape of the curves near the minimum nor

Rewriting the weight equation in non-dimensional form

[illegible]

[illegible]

[illegible]

The first term on the right may be rewritten as

[illegible]

or as

[illegible]

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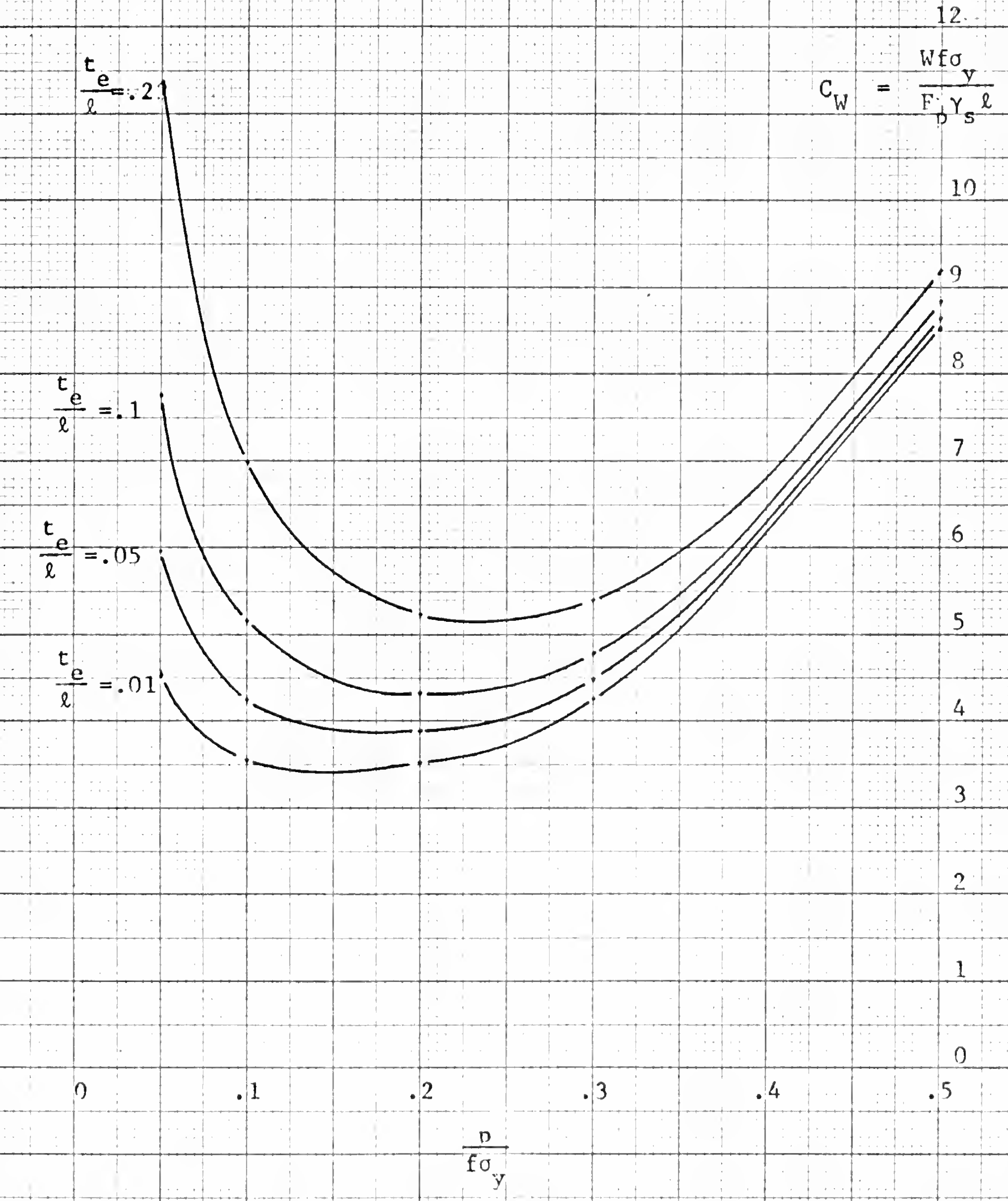
FIGURE VI

OPTIMIZATION CURVES FOR HYDRAULIC PISTON AND CYLINDER



about the behavior of the variables in general. For this reason it was decided to plot the equation from which the following general conclusions may be drawn.

a) Curves are flat in vicinity of minimums.

b) The minimum value decreases with decreasing t_e/ℓ, but t_e/ℓ is determined by F_p, $f\sigma_y$, and ℓ. That is, it is determined by the application except for the value of $f\sigma_y$.

c) Can choose $f\sigma_y$ by taking a pressure, say 5000 psi, and then trying to get, say, 5 or 10% up from the minimumpoint. Then plug this into the t_e/ℓ equation. Note that increasing $f\sigma_y$ in this way from that corresponding to the min $p/f\sigma_y$ value will decrease t_e/ℓ but its effect is very weak.

d) As $f\sigma_y$ goes down, the actual weight goes up proportionally because of the $f\sigma_y$ term in the non-dimensional weight coefficient. However, generally desire σ_y as low as possible primarily for cost considerations which, although alluded to here, will not be investigated.

e) Note the effect of increasing ℓ is to lower t_e/ℓ but this effect is not as large as its effect on C_w which causes weight to go up proportionally with ℓ.

f) The effect of increasing F_p is to cause t_e/ℓ to increase weakly and C_w to increase linearly. Therefore, a proportional increase in F_p will cause more increase in weight than will the same increase in length.

3.2.1.4 Hydraulic Pumps

Design of hydraulic pumps is an extremely complicated business which is heavily dependent on empirical considerations particularly in the area about the behavior of the variables in general. For this reason it was decided to plot the equation f on which the following general conclusions may be drawn.

a) Curves are flat in vicinity of minimum.

b) The minimum value decreases with decreasing t_o/λ but t_o/λ is determined by F_p, $f\sigma_y$ and λ. That is, it is determined by the specification except for the value of $f\sigma_y$.

c) Can choose $f\sigma_y$ by making pressure [illegible] and then trying to get [illegible] 5 or [illegible] the minimum. Then plug it into the t_o/λ equation. Note that increasing $f\sigma_y$ in this way from that corresponding to the minimum value will decrease t_o/λ but its effect is very weak.

d) As $f\sigma_y$ goes down the actual weight goes up proportionally because of the $f\sigma_y$ term in the non-dimensional weight coefficient. However, generally desire σ_y as low as possible primarily for cost considerations which, although included on here, will not be investigated.

e) Note the effect of increasing λ is to lower t_o/λ but this effect is not as large as its effect on C_w which causes weight to go up proportionally with λ.

f) The effect of increasing F_p is to cause t_o/λ to increase and C_w to increase linearly. [illegible] proportional increase in [illegible] will cause more increase in weight than will the same increase in [illegible]

3.2.1.4 Hydraulic Pumps

Design of hydraulic pumps is an extremely complicated matter which is heavily dependent on empirical considerations particularly in the area of estimating wear life under different applications. It is actually a science unto itself, and, as such, it is beyond the scope of this investigation.

The approach used here was to obtain data from several manufactures and then plot this in the form of weight versus flow rate for several pressures. For a given pressure, weight goes up linearly with flow rate Q. The points on these lines which meet the requirements of this application can be located. Then with some luck, an analytic approximation might be obtainable for the curve through these points. Initial results obtained by plotting data on Vickers pumps[13] in Figure (VII) were encouraging. It is to be emphasized that the curve drawn does not represent the weights that would be obtained if existing equipment were used. However, an analytic expression is desired to facilitate the system weight analysis, and the curve does represent the weights that could be expected if pumps were custom built for this application using the same design parameters as the existing pumps.

As soon as the data from other manufacturers[14,15,16] was plotted, also shown in Fig. (VII), it became apparent that there was far more weight dependence on manufacturer than on pressure. Evidently improvements in technology have permitted the new family of high speed 5000 psi pumps to be designed with considerably lower weights. Figure (VII) suggests that a more realistic approach to this problem would be to locate those existing pumps which have the required capacity for this application, and then consider weight as a constant independent of pressure in the range of applicability of those pumps. It is clear from of estimating wear life under different applications. It is actually a science unto itself, and as such, it is beyond the scope of this investigation.

The approach used here was to obtain data from several manufacturers and then plot this in the form of weight versus flow rate for several pressures. For a given pressure, weight goes up linearly with flow rate Q. The points on these lines which meet the requirements of this application can be located. Then with some luck an analytic approximation might be obtainable for the curve through these points. Initial results obtained by plotting data on Vickers pumps[13] in Figure (VII) were encouraging. It is to be emphasized that the curve drawn does not represent the weights that would be obtained if existing equipment were used. However, an analytic expression is desired to facilitate the system weight analysis, and the curve does represent the weights that could be expected if pumps were custom built for this application using the same design parameters as the existing pumps.

As soon as the data from other manufacturers[14,15,16] were plotted, also shown in Fig. (VII), it became apparent that there was far more weight dependence on manufacturer than on pressure. Evidently improvements in technology have permitted the new family of high speed 3000 psi pumps to be designed with considerably lower weights. Figure (VII) suggests that a more realistic approach to this problem would be to locate those existing pumps which have the required capacity for the application, and then consider weight as a constant independent of pressure in the range of applicability of those pumps. It is clear from

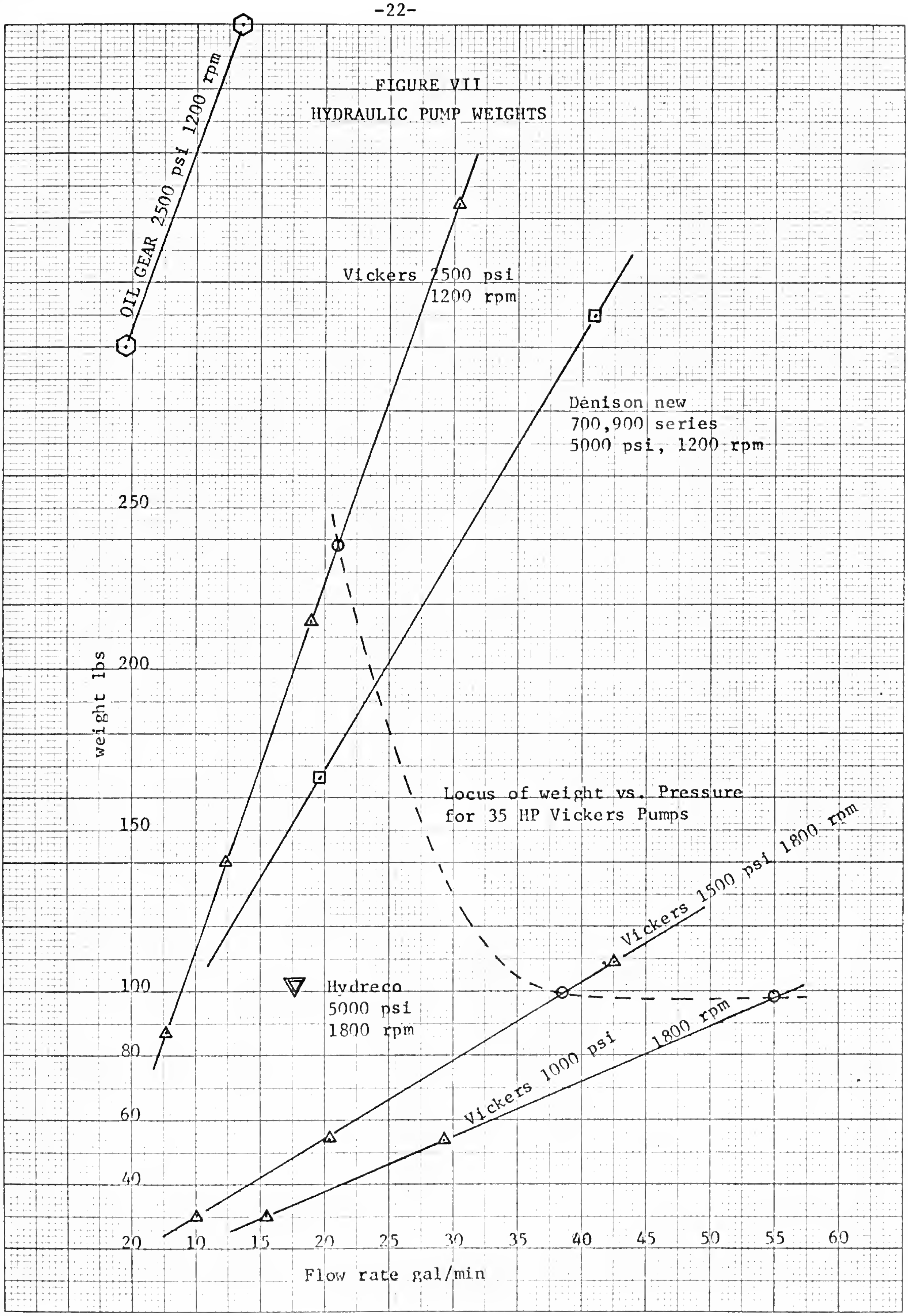


FIGURE VII
HYDRAULIC PUMP WEIGHTS
OIL GEAR 2500 psi 1200 rpm
Vickers 2500 psi
1200 rpm
Denison new
700,900 series
5000 psi, 1200 rpm
Locus of weight vs. Pressure
for 35 HP Vickers Pumps
Vickers 1500 psi 1800 rpm
Hydreco
5000 psi
1800 rpm
Vickers 1000 psi 1800 rpm
weight lbs
250
200
150
100
80
60
40
20
10
15
20
25
30
35
40
45
50
55
60
Flow rate gal/min

Fig. (VII) that no penalty need be incurred in pump weight in going to 5000 psi pressures.

3.2.1.5 Tiller and Linkage, Rapson Slide, and Rack and Pinion

The equations governing the operation of these devices are derived in Appendix IV using the notation shown in Fig. VIII. It is of interest to compare the length of travel for these three devices. For maximum rudder angle $\theta_m = 35^\circ$, travel is equal to

$$\text{Tiller and linkage} = \frac{2T}{F_p} (.7) \qquad (25)$$

$$\text{Rapson slide} = \frac{2T}{F_p} (.47) \qquad (26)$$

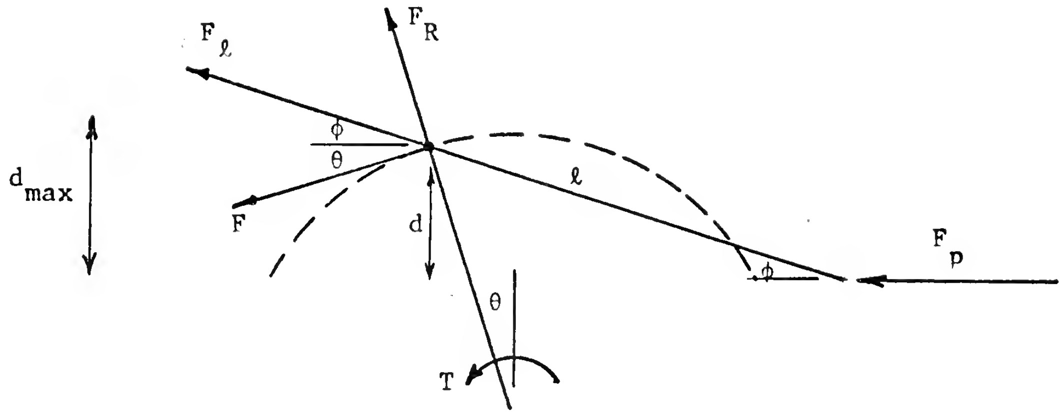
$$\text{Rack and pinion} = \frac{2T}{F_p} (1.221) \qquad (27)$$

It can be seen that the travel of the rapson slide is markedly shorter for the same maximum torque T and piston force F_p. The cylinder length ℓ equals the travel, and it has already been noted that decreasing ℓ will decrease cylinder weight. For this reason the rapson slide is chosen for further analysis. In addition to the above consideration, the tiller and linkage and rack and pinion by themselves are equal to or heavier in weight than the rapson slide. The details of this are given at the end of Appendix VI.

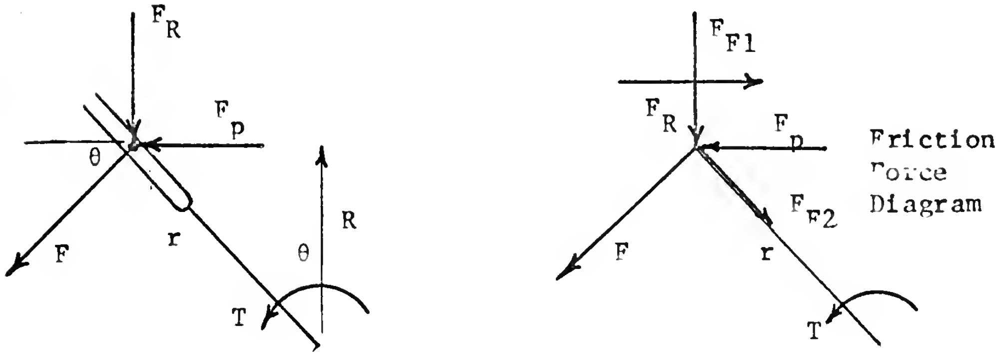
Although the weight of a rapson slide can be calculated, it is important to know what parameters should be varied in order to yield the most meaningful results. This requires a careful evaluation of the problem

FIGURE VIII

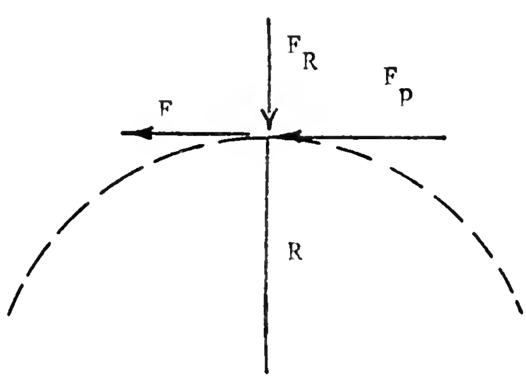
ROTARY MECHANISM NOTATION



Force Diagram for Tiller and Linkage



Force Diagram for Rapson Slide



Force Diagram for Rack and Pinion

from the viewpoint of the system as a whole.

3.2.1.6 Problem Definition and Solution

The weight optimization problem for the complete system must now be accurately defined so that the procedure required to perform the optimization can be determined.

Divide weight optimization into 3 parts: (a) crosshead and linkage group, (b) cylinder and piston, (c) pump. Consider T as given and hence constant for the problem.

(a) Weight of crosshead and linkage is a function of piston force, travel length and crosshead radius

$$W_{(a)} = f(F_p, \ell, R) \text{(also others as } f\sigma_y\text{, rudder stock diameter)} \quad (28)$$

but $$(F_p)(\ell) = \text{constant} = 2T \tan\theta_{max} \cos^2 \theta_{max} \quad (29)$$

and $$\ell = R2 \tan\theta_{max} \quad (30)$$

therefore $$W_a = f'(R, f\sigma_y) \quad (31)$$

Note that it can be made independent of system pressure.

(b) Weight of cylinder and piston is a function of F_p, ℓ, p, $f\sigma_y$, r_i, k, and t_e.

$$t_e = f(F_p, f\sigma_y) = f(\ell, f\sigma_y) = f(R, f\sigma_y) \quad (32)$$

From the viewpoint of the system as a whole.

3.2.1.6 Problem Definition and Solution

The weight optimization problem for the complete system [illegible] be accurately defined [illegible] optimization can be determined.

Divide weight optimization into [illegible] parts [illegible] group, (b) cylinder and piston, [illegible] constant for the problem.

(a) Weight of crosshead and linkage [illegible] travel length and crosshead [illegible]

$W_{(a)} = f(P_p, \ell, ?)$ [illegible]

but $(P_p)(?)$ = constant [illegible] $_{max}$ [illegible]

and $\ell = K\ell$ [illegible] $_{max}$

therefore $W_a = f'(\ell, f_y)$

Note that it can be made independent of system [illegible]

(b) Weight of cylinder and piston is [illegible] k, and t_e.

t_p = [illegible] f_y) = $f(\ell, t c_y)$ = [illegible]

$$k = f(p, f\sigma_y) \tag{33}$$

$$F_p = \text{const.} \tag{34}$$

$$\ell = f(R) \tag{35}$$

$$F_p = f(p, r_i) = p\pi r_i^2 \tag{36}$$

$$\therefore W_{(b)} = f(R, p, f\sigma_y) \tag{37}$$

(c) Weight of pumps= $f(p,Q)$

but $p \times Q$ = constant. Therefore $W_c = f(p)$ or $f(Q)$ (38)

and the total weight, $W_t = W_{(a)}+W_{(b)}+W_{(c)}$ is a function of two <u>independent</u> variables R and p.

$$W_t(R,p) = W_a(R)+W_b(R,p)+W_c(p) \tag{39}$$

In order to optimize, the following partial differential equations must be satisfied.[17]

$$\frac{\partial W_b}{\partial p} \div \frac{\partial W_c}{\partial p} = 0 \tag{40}$$

$$\frac{\partial W_a}{\partial R} + \frac{\partial W_b}{\partial R} = 0 \tag{41}$$

If it is recalled that pump weight can be considered constant, the solution of equation (40) reduces to determining the optimum pressure for the piston and cylinder. This is most easily done by selection of an

optimum $\frac{p}{f\sigma_y}$ from Fig. VI as discussed in section 1.2.2.

In order to solve equation (41), the weight equation for the piston and cylinder must be converted from the variable ℓ to the variable R, the nominal radius of the rapson slide. In other words, the general equation for cylinder weight must be specialized to the parameters involved in this particular application.

Going back to equation (24) for the weight of the piston and cylinder, the first term on the right-hand side is:

$$[2(1 - \frac{\gamma_o}{\gamma_s}) \sqrt{\frac{\alpha F_p}{\pi f \sigma_y \ell^2}} + \frac{\gamma_o}{\gamma_s}] \tag{24b}$$

but from Appendix IV

$$F_p \ell = \frac{T}{R} (\cos^2\theta_m + .2 \sin\theta_m \cos\theta_m) 2R \tan\theta_m \tag{42}$$

Define $$(\cos^2\theta_m + .2 \sin\theta_m \cos\theta_m) = \psi \tag{43}$$

Then $$F_p = \frac{2T\psi\tan\theta_m}{\ell} \tag{44}$$

Substitute into the above to obtain

$$[2(1 - \frac{\gamma_o}{\gamma_s}) \sqrt{\frac{2T\psi\tan\theta_m}{\pi f \sigma_y \ell^3}} + \frac{\gamma_o}{\gamma_s}] \tag{45}$$

$$\ell^3 = 8R^3\tan^3\theta_m \quad \text{(also from Appendix IV)} \tag{46}$$

Now have $$[2(1 - \frac{\gamma_o}{\gamma_s}) \frac{1}{\tan\theta_m} \sqrt{\frac{\psi\alpha T}{\pi f \sigma_y R^3}} + \frac{\gamma_o}{\gamma_s}] \tag{47}$$

The non-dimensional group here is $\dfrac{T}{f\sigma_y R^3}$.

The whole expression for cylinder and piston weight is

$$\frac{Wf\sigma_y}{F_p \ell \gamma_s} = \frac{f\sigma_y}{p}\left[\left(1 - \frac{\gamma_o}{\gamma_s}\right)\frac{1}{\tan\theta_m}\sqrt{\frac{\alpha\psi T}{\pi f\sigma_y R^3}} + \frac{\gamma_o}{\gamma_s}\right] + \frac{f\sigma_y}{p}\left[\frac{3+\sqrt{\dfrac{4f^2\sigma_y^2}{p^2} - 3}}{\dfrac{f^2\sigma_y^2}{p^2} - 3}\right] \tag{48}$$

Of course, the left-hand side can be written in terms of the given torque as

$$\frac{Wf\sigma_y}{2T\psi\tan\theta_m\gamma_s} \tag{49}$$

This equation is plotted in Fig. (IX) for several variations of the non-dimensional parameters.

In order to solve the partial differential equation (41) with respect to R, it is necessary to determine the variation in weight of the rapson slide with R. Each component of this system can be examined qualitatively and the expected trend of its weight variation with R predicted. However, since the weights of these components often vary in opposite directions, the net sum of the variations is not apparent. It is therefore necessary to actually calculate the weight of several possible rapson slides for different R values. These calculations are in Appendix V and the results shown on Fig(IX) . For the purposes of these calculations, a design was chosen which would simplify somewhat the calculations, incorporate the most important features desired in good design, and still give meaningful results.

FIGURE IX

OPTIMIZATION CURVES OF A PISTON AND CYLINDER FOR A RAPSON SLIDE

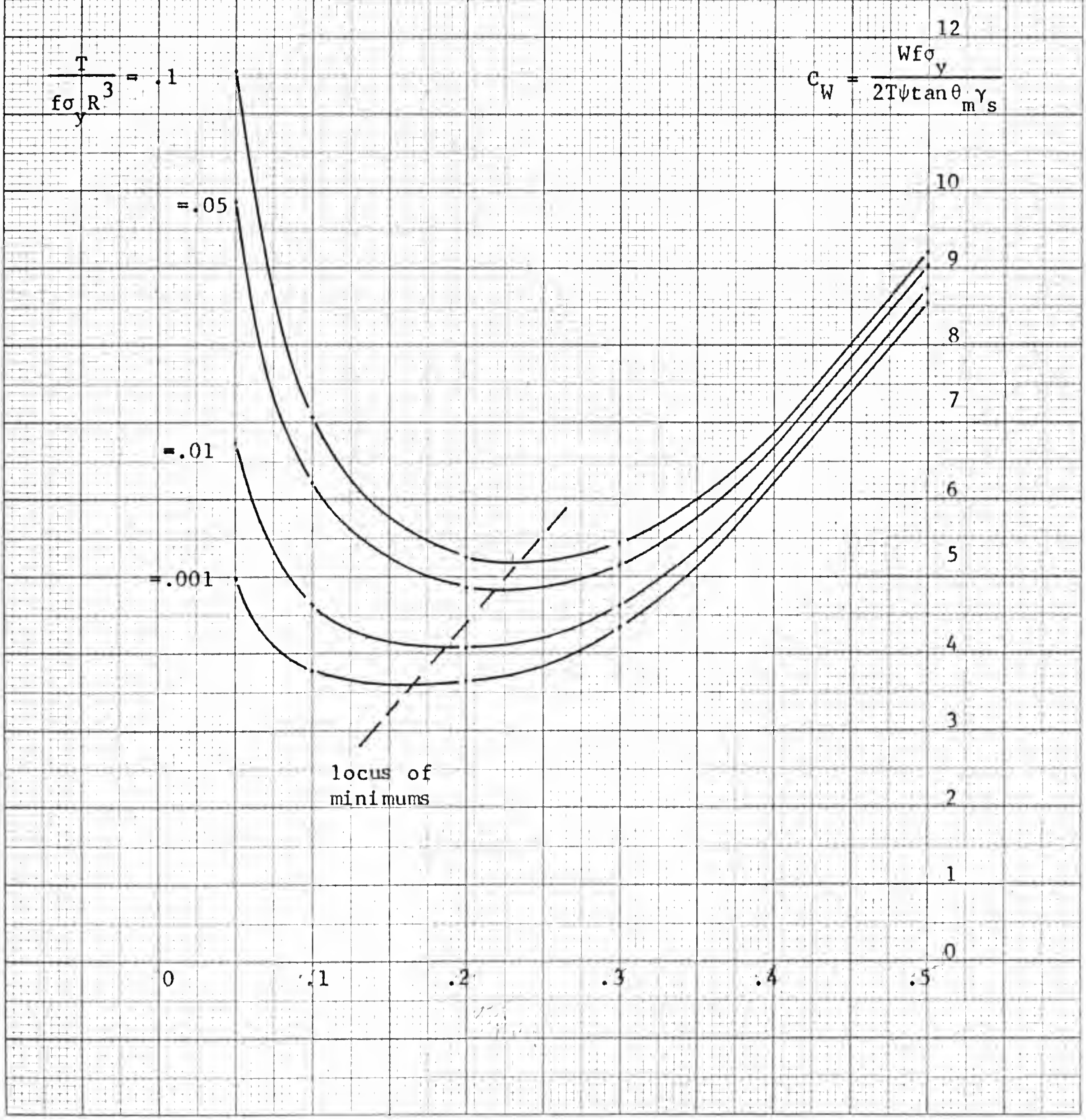
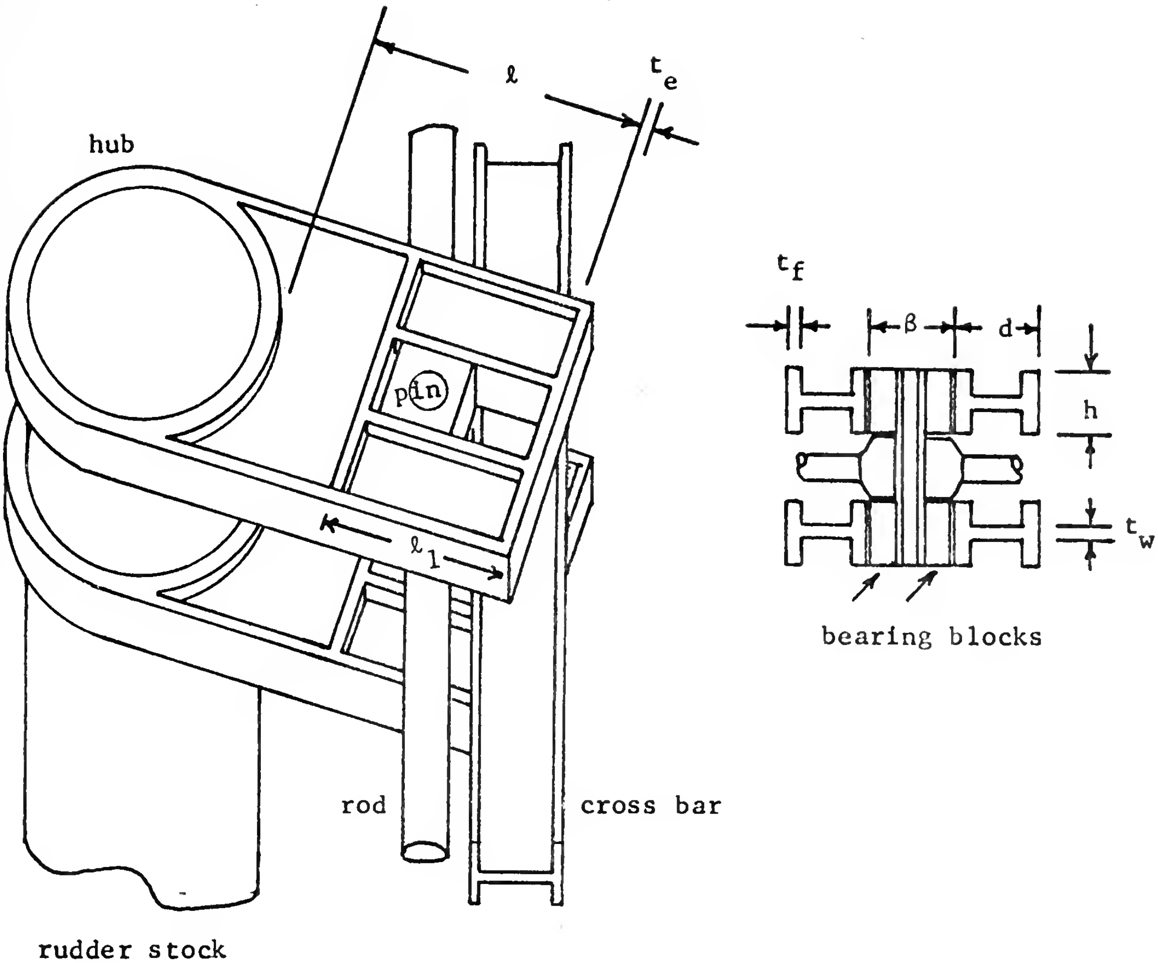
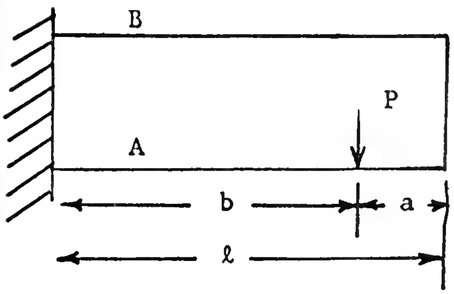


FIGURE X



Rapson Slide Arrangement

Beam Configuration



Beam Formulas

Maximum loadings occur in beam B

$$\text{Beam B shear} = \frac{P_b^2}{2^3}(a+2\ell)$$

$$\text{Mat root} = \frac{P_b^2}{2^2}(a+2\ell)$$

This design is shown in Fig. (XI). Some recent designs[2] have used a single unit tiller with the bearing pads located externally in a yoke and collar device as opposed to the slot design used here. (See Fig. (I)). This system has two disadvantages. The first is that the tiller arm must have a narrow depth which means that it is not a very efficient type of beam to withstand the high bending moments imposed upon it. The second is that this arrangement requires an inordinately large yoke and collar.

Use of two split Tiller arms appears to provide the best way of balancing the forces on the crosshead pin and the bearing blocks. End plates to carry the load to the otherwise unloaded arm are considered essential to the design. Transverse stiffners appear to be the most efficient way of supplying the required rigidity to the flange which serves as the bearing surface. It is lighter in weight than merely thickening that flange.

A cross bar is used to take up the transverse reaction force (F_R) of the slide. This allows the use of an I beam shape to carry the bending load rather than the less efficient circular cross section of the rod. This also allows the rod and yoke to be designed for compressive stress and column buckling only. One of the advantages of this is that it permits the use of smaller diameter pistons and cylinders associated with higher pressures without having to restrict this diameter in order to get a large enough ram to carry the bending moment produced by F_R. Also in higher pressure units, the seals become a critical item, and it is desirable in their design to have as little deflection and side loading

as possible. One further advantage is that this permits the use of double acting pistons in the cylinder. Curves similar to Fig. (IX) may be calculated for the double acting piston and the weights compared. It is expected that this would lead to a lighter design. Although the added complexity of this arrangement is probably unwarranted in the low torque range, it appears well worth the effort in large units. In actual practice a tapered beam design would be used. That is the depth of the beam would be reduced at the end. This has the desirable effect of reducing the clearances so that shorter rods can be used. Although this is standard in most slotted designs it was not used here because of computational complexity.

Inspection of Fig.(XI) leads to the following observations and conclusions.

(a) Total percentage charge is small (8.3%).

(b) Weight decreases irregularly with R.to a shallow minimum at about R = 40.

(c) At about R = 40, unsupported lengths of component parts get long enough to get into buckling instabilities, hence this weight is not obtainable in actuality. From this point on, weights will begin to rise due to stiffening required to prevent buckling (in addition to that shown).

(d) Minimum point is not clear and in general will depend on the details of the particular design.

(e) Weight of piston and cylinder decreases with increasing R (from Fig. IX). Hence the optimum from a weight point-of-view is at that R where heavy additional stiffeners are required to prevent buckling in members.

as possible. The further advantage [illegible] double acting pistons in the cylinder. [illegible] may be calculated for the double acting [illegible]. It is expected that this would lead to [illegible] added complexity of this arrangement [illegible] low torque range, it appears [illegible] actual practice a tapered beam [illegible] of the beam would be [illegible] of reducing the [illegible] this is standard [illegible] of computational [illegible].

Inspection of the (12) [illegible] conclusions.

(a) Total [illegible]

(b) [illegible] about R = 4 [illegible].

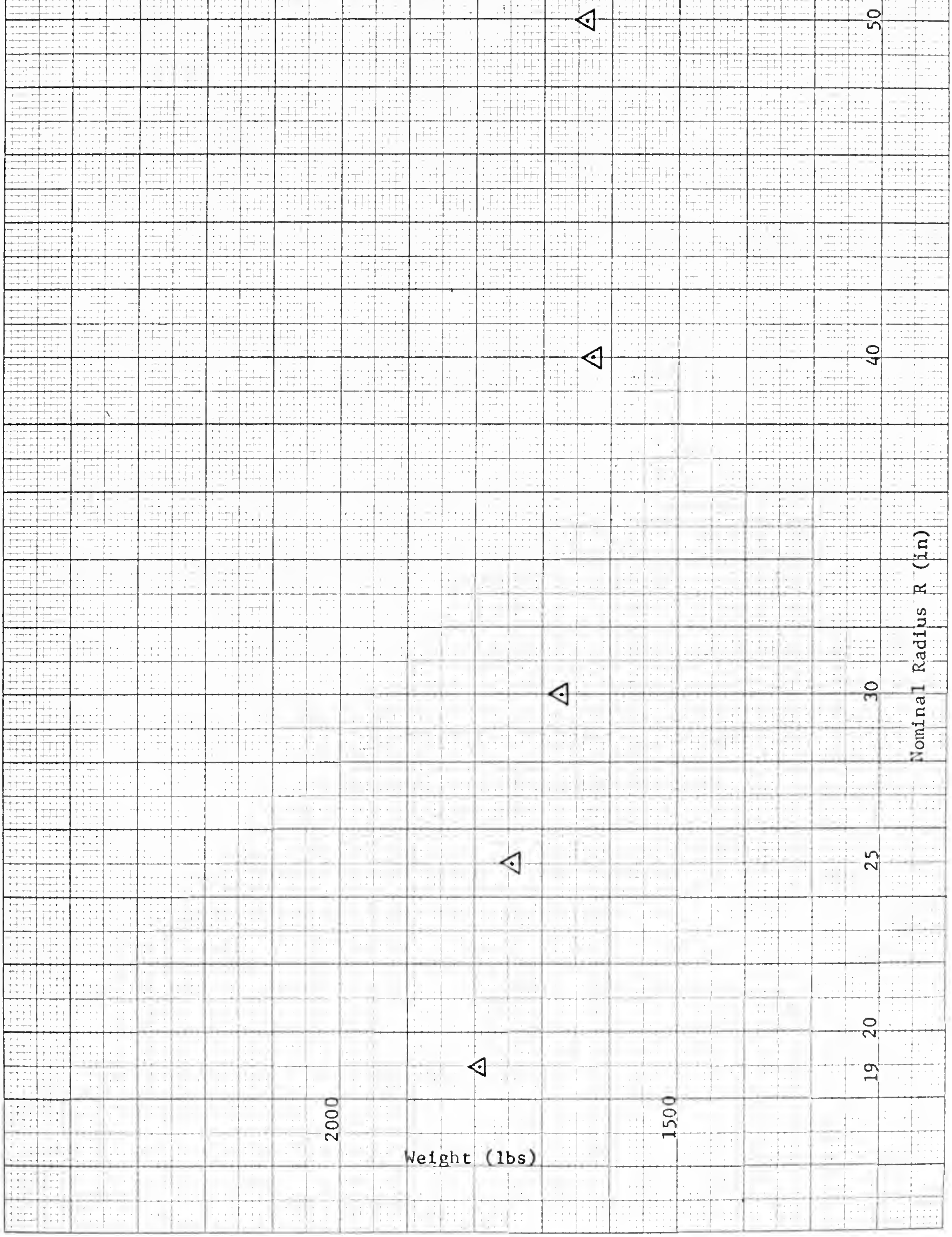
(c) [illegible] long enough [illegible] obtainable in actuality. [illegible] due to stiffening required to [illegible]

(d) Minimum point is not [illegible] details of the particular design.

(e) Weight of [illegible] (from Fig. 13). [illegible] that R where heavy additional stiffening [illegible] in members.

FIGURE XI

RAPSON SLIDE WEIGHTS



(f) Maximum weight reduction that can be obtained in going from minimum R (most compact design) to optimum R appears to be only about 10%.

(g) Weights will go down if rudder stock is larger than 24". Onset of buckling will occur at a larger R. Reduction in weight by optimizing will be a smaller percentage. However, overall weight will be less.

3.2.1.7 Summary of Weights

The weights for the piston and cylinders are worked out from Fig. XI in Appendix V. The weight of the drive motor and hydraulic pump are also calculated and the results of those calculations are summarized below.

	R = 19	R = 25	R = 30	R = 40	R = 50
Rapson slide	1791	1743	1672	1627	1637
Piston and cylinder	986	864	840	790	768
Hydraulic pump, etc.	310	310	310	310	310
Drive motor	360	360	360	360	360
Totals	3,447 lbs	3,277 lbs	3,182 lbs	3,087 lbs	3,075 lbs

The above calculation was carried out using a value of $f\sigma_y$ = 17,500 psi. If a steel with a value of $f\sigma_y$ = 35,000 psi were used, the following piston and cylinder weights result.

R = 19	R = 25	R = 30	R = 40	R = 50
497 lbs	466 lbs	437 lbs	396 lbs	384 lbs

(f) Maximum weight reduction that can be obtained in going from minimum R (most compact design) to optimum R appears to be only about 10%.

(g) Weights will go down if rudder stock is larger than 2". Onset of buckling will occur at a larger L. Reduction in weight by optimizin; will be a smaller percentage. However, overall weight will be less.

3.2.1.7 <u>Summary of Weights</u>

The weights for the piston and cylinders are worked out from Fig. [illegible] in Appendix V. The weight of the drive motor and hydraulic pump are also calculated and the results of these calculations are summarized below.

	R = 15	R = 25	R = 30	R = 40	R = 50
Rapson slide	1791	1743	1672	1627	1657
Piston and cylinder	986	964	840	790	750
Hydraulic pump, etc.	310	310	310	310	310
Drive motor	360	360	360	360	360
Totals	3,447 lbs	3,377 lbs	3,182 lbs	3,087 lbs	3,075 lbs

The above calculation was carried out using a value of σ_y = 17,500 psi. If a steel with a value of σ_y = 35,000 psi were used, the following piston and cylinder weights result.

R = 15	R = 25	R = 30	R = 40	R = 50
497 lbs	455 lbs	437 lbs	396 lbs	[illegible] lbs

These very low values for the piston and cylinder result from the high pressures used. For instance, if pressure is doubled, the piston area is halved, but the diameter is reduced by a factor of 4. This rapid reduction in diameter produces the dramatically light weights seen above. The higher pressure does not result in very much thicker tube walls. This is because the thickness equation works out to be a fixed percentage of diameter for a given pressure. Therefore, if diameter is decreased, there is a reduction in thickness to offset the higher percentage due to the higher pressure. The rapson slide is seen to be quite an efficient device resulting in only a twenty-five horsepower drive motor.

3.2.2 ROTARY VANE ACTUATOR

3.2.2.0 General

The rotary vane actuator steering engine has many attractive features, foremost among which is that it is the simplest mechanical device that will do the job. Along with this inherent simplicity goes ruggedness, shock resistance, compactness, and ease of installation, maintenance, and operation. Its only major defect is that it is extremely difficult to seal adequately and as a result, these units usually have high leakage rates. There is such a long length of periphery around the vanes that seals with a leakage rate per unit length that would be acceptable in other hydraulic devices produce a total leakage rate that is too large in the rotary vane actuator. On the other hand, making the seals too tight in an effort to reduce the leakage, produces large friction forces which give the unit high "breakout" or no load starting torque and low

These very low values for the lateral cylinder [illegible] in the high pressures used. For instance, if pressure is doubled, the surface area is halved, but the diameter is reduced by a factor of [illegible]. This rapid reduction in diameter produces the dramatically low [illegible] seen above. The higher pressure does not result in very much thicker tube walls. This is because the thickness required tends to be a fixed percentage of diameter for a given pressure. Therefore, as diameter is decreased, there is a reduction in thickness to offset the higher percentage due to the higher pressure. The [illegible] is [illegible] to be quite an efficient [illegible] resulting in only a [illegible] horsepower drive motor.

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Leakage flow in positive displacement hydraulic components can generally be divided into two types of fluid flow. The first is flow between parallel flat plates for which the flow rate is directly proportional to pressure. The second is orifice flow in which the flow rate can be expected to vary roughly as the square root of pressure.[19] The leakage in a rotary vane actuator is a combination of these two types, and so it would be expected that the flow rate would vary as the pressure raised to some power between a half and one. However, recent efforts in the industry have been able to reduce the leakage well below this, and in the case shown in Fig. (XII), the leakage rate can be seen to approach a function of pressure raised to only the .3 power. One of the methods of achieving this reduction in leakage is to design the seals so that as the pressure rises, it acts to close the clearance distance through which the leakage can flow. This means that at low pressures the leakage rate increases faster than at high pressures, but this is acceptable because at these low pressures the total leakage is still within reasonable bounds. Another advantage of this type of seal is that it reduces the no load starting torque.

It can be seen from Fig. (XIII) that at higher pressures the leakage rate is more accurately represented in terms of exponentionals. It is to be expected that in view of the nature of the problem, prediction of efficiency under operation.[14] Thus it can be seen that the seals are the critical part of the rotary vane unit's design, and in fact, much of the recent popularity of the device is attributable to improvements/ progress in this area.

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FIGURE XII

LOG-LOG PLOT OF LEAKAGE RATE

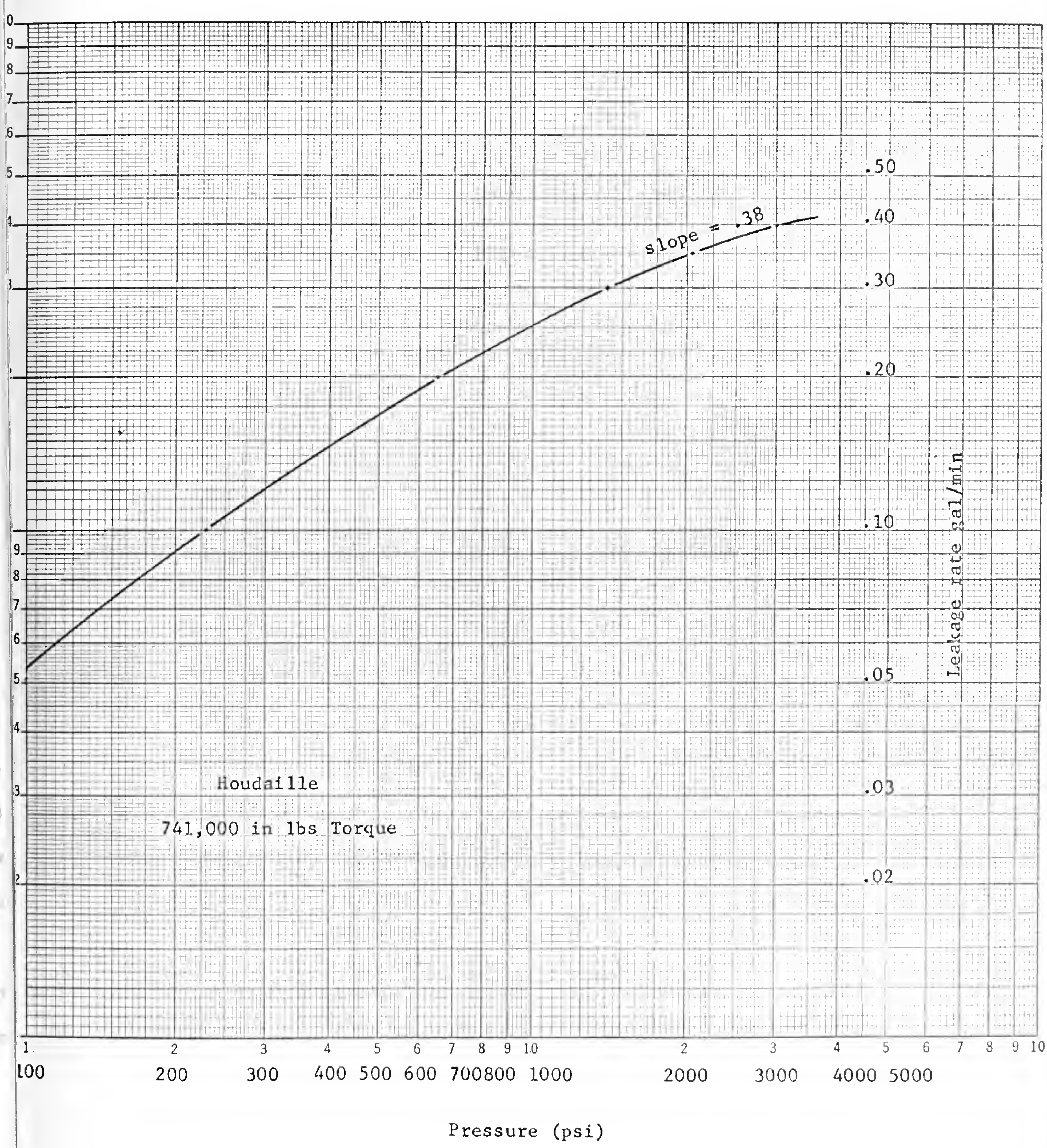
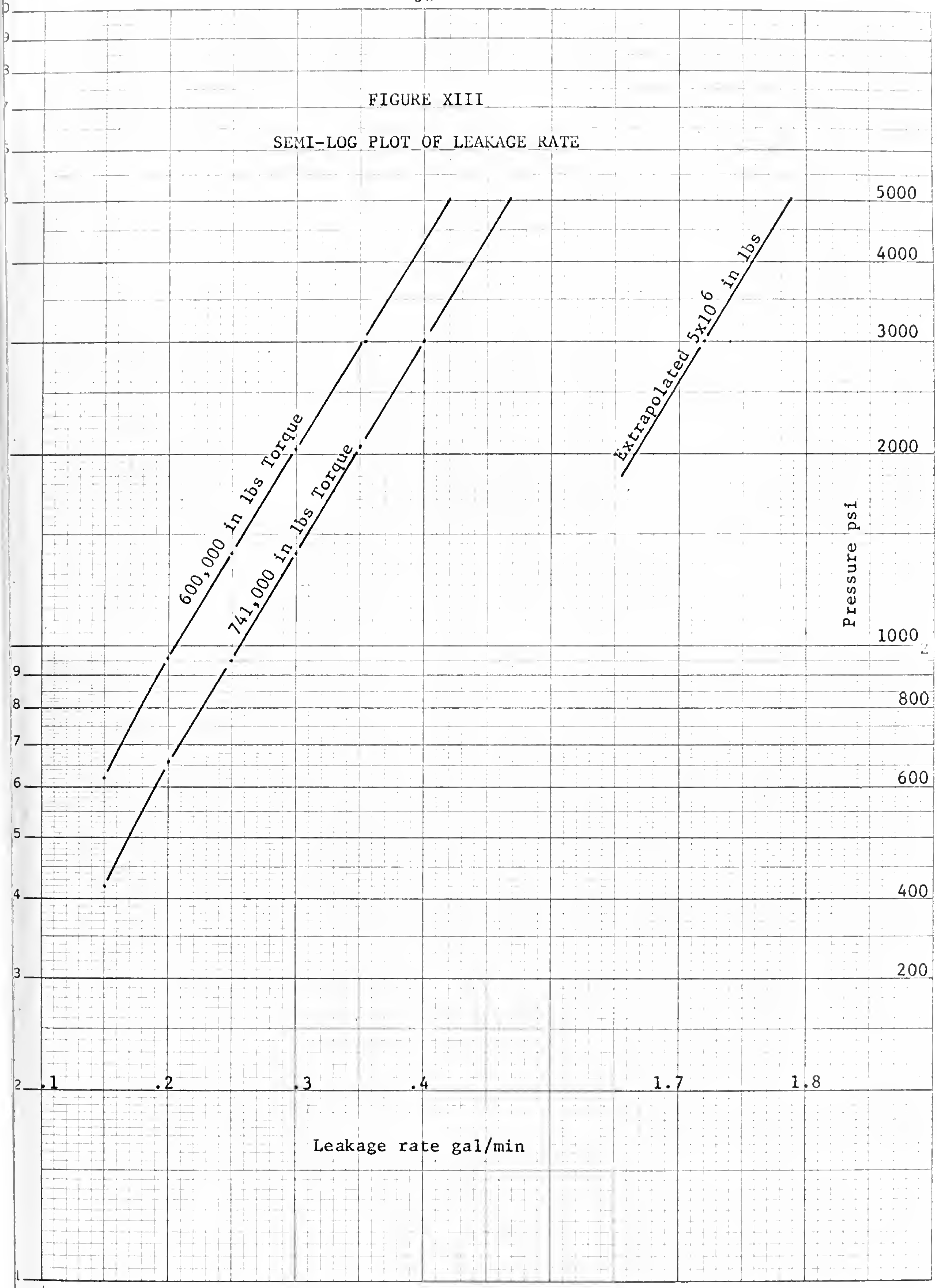


FIGURE XIII

SEMI-LOG PLOT OF LEAKAGE RATE



leakage rates must continue to rely heavily on empirical data. There appears to be no technical reason that maximum operating pressures cannot be extended to the 5000 psi range should this ever prove desirable. It also seems reasonable to expect that the leakage characteristics could be obtained by extrapolation from lower pressure ranges. There is also no technical reason that these actuators cannot be built in the several million inch pound torque range. In these higher torque ranges some improvement in leakage rate might be expected. This is because one of the major factors in leakage control is manufacturing tolerances of the parts. These remain substantially the same even though overall size of the unit increases. Thus the leakage would be a lower percentage for the larger unit.[19] In addition, there is more space available in the larger unit permitting the installation of more elaborate seals and other design refinements not practicable in the smaller units.

There are other ramifications of the leakage problem which merit attention. The power consumed in pumping the leakage through a rotary vane actuator is the product of the pressure times the volume rate of leakage flow. Thus the power loss in leakage rises more rapidly than pressure even though the leakage itself increases more slowly. The application engineer is thus faced with the problem of deciding when the power loss is "too much". It is a very difficult thing to say when it is "better" to install a larger lower pressure unit with lower leakage than a smaller higher pressure unit with its higher power loss.

There are several view points which can be taken in interpreting what is meant by "better". From a weight point of view, it is probably leakage rates must continue to rely heavily on empirical data. There appears to be no technical reason that maximum operating pressure cannot be extended to the 5000 psi range should this ever prove desirable. It also seems reasonable to expect that the leakage characteristics could be obtained by extrapolation from lower pressure ranges. There is also no technical reason that these actuators cannot be built in the several million inch pound torque range. In these higher torque ranges some improvement in leakage rate might be expected. This is because one of the major factors in leakage control is manufacturing tolerances of the parts. These remain substantially the same even though overall size of the unit increases. Thus the leakage would be a lower percentage for the larger unit.[19] In addition, there is more space available in the larger unit permitting the installation of more elaborate seals and other design refinements not practicable in the smaller units.

There are other ramifications of the leakage problem which merit attention. The power consumed in pumping oil leakage through a rotary vane actuator is the product of the pressure times the volume rate of leakage flow. Thus the power loss in leakage rises very rapidly with pressure even though the leakage itself increases more slowly. The application engineer is thus faced with the problem of deciding when the power loss is "too much". It is a very difficult thing to say when it is "better" to install a larger lower pressure unit with lower loss than a smaller higher pressure unit with its higher power loss.

There are several view points which can be taken in interpreting what is meant by "better". From a design point of view, it is generally "better" to go to higher pressures and accept the lower overall efficiencies of the unit. This is because the hydraulic pump and its motor are generally considerably smaller than the actuator. Another consideration is the expected operating pattern that the unit will see. If it will only be required to develop maximum torque a relatively small percentage of its lifetime, then the high power losses become an occasional occurrence and may be more easily tolerated.

3.2.2.1 Analysis

It is in order now to proceed with the mathematical analysis starting with the bending moment at the base of the vanes which can be easily calculated. From this the tension and shear stresses can be calculated which can then be resolved into the principal stresses. Then if the Hencky-Von Mises shear energy strength theory is used, an expression for the maximum permissible stress in the vane root can be calculated. Nomenclature is given in Fig. (IXX).

If $\frac{D}{t} < 1.5$ and a fillet of radius $\approx t$ is used, there will be less than 7% stress concentration at the root.[20]

Shear stress at the root is $$\tau_{xy} = \frac{p\ell w}{wt} = p\left(\frac{\ell}{t}\right) \tag{50}$$

Stresses at top fiber are $$\sigma_x = 3p\left(\frac{\ell}{t}\right)^2, \quad \sigma_y = -p$$

these stresses are now converted to principal stresses "better" to go to higher pressures and accept the lower overall efficiencies of the unit. This is because the hydraulic pump and motor are generally considered smaller than the actuator. Another consideration is the expected operating pattern that the unit will see. If it will only be required to develop maximum torque a relatively small percentage of its lifetime, then the high pressure losses become an occasional occurrence and may be more easily tolerated.

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If $\frac{b}{t} < 1.5$ and a fillet of radius [illegible] is used, there will be less than 7% stress concentration at the root.[illegible]

Shear stress at the root is

$$\tau_{xy} = [illegible] \qquad (50)$$

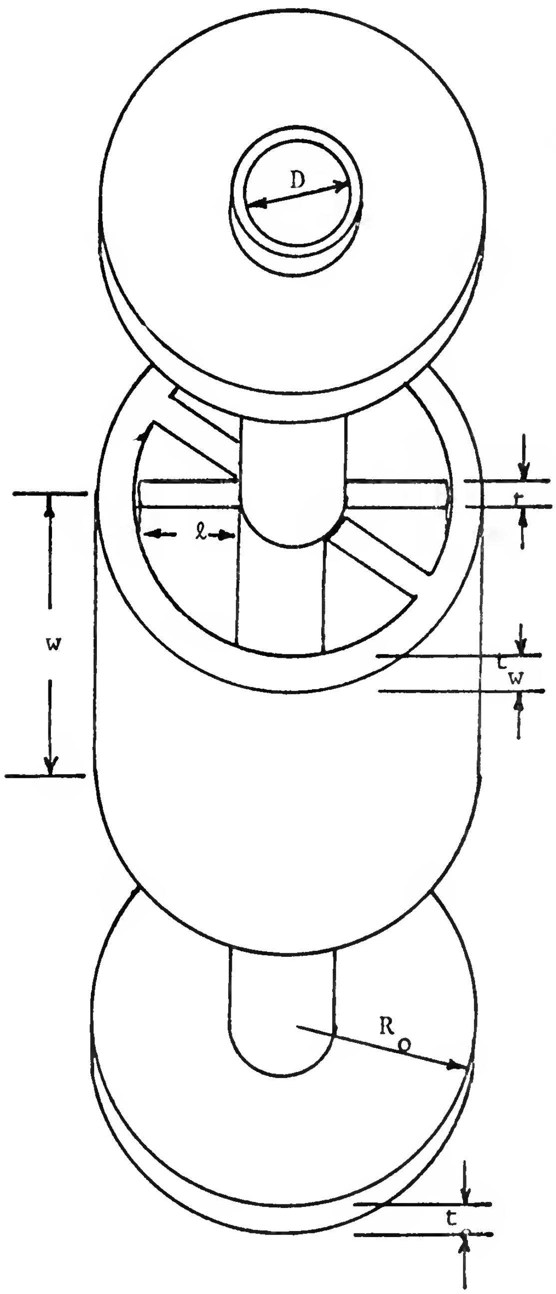
Stresses at top fiber are

$$\sigma_x = 3b(\tfrac{[illegible]}{t})^2 [illegible] \qquad \sigma_y = [illegible]$$

these stresses are now converted to principal stresses

FIGURE IXX

ROTARY VANE ACTUATOR NOMENCLATURE



$t_w = R_o - R_i$

$R_o = KR_i$

$t_w = R_i - (K-1)$

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\frac{\sigma_x-\sigma_y}{2}^2 + (\tau_{xy})^2}$$

$$= \frac{3p(\frac{\ell}{t})^2 - p}{2} + \sqrt{[\frac{3p(\frac{\ell}{t})^2+p}{2}] + (p\,\frac{\ell}{t})^2} \qquad (51)$$

$$\sigma_1 = \frac{p}{2}\,[3(\frac{\ell}{t})^2-1) + \frac{p}{2}\sqrt{(3(\frac{\ell}{t})^2+1)^2+4(\frac{\ell}{t})^2\,]} \qquad (52)$$

$$\sigma_1 = \frac{p}{2}\,[3(\frac{\ell}{t})^2-1 + \sqrt{9(\frac{\ell}{t})^4+10(\frac{\ell}{t})^2+1]} \qquad (53)$$

$$\sigma_2 = \frac{p}{2}\,[3(\frac{\ell}{t})^2-1- \sqrt{9(\frac{\ell}{t})^4+10(\frac{\ell}{t})^2+1]} \qquad (54)$$

Using the maximum shear energy theory (Hencky-Von Mises)

$$\sigma_{max}^2 = \sigma_1^2-\sigma_1\sigma_2+\sigma_2^2 \qquad (55)$$

$$f^2\sigma_y^2 = \frac{p^2}{4}\,[3(\frac{\ell}{t})^2-1)^2+3(9(\frac{\ell}{t})^4+10(\frac{\ell}{t})^2+1)]$$

$$= \frac{p^2}{4}\,[9(\frac{\ell}{t})^4-6(\frac{\ell}{t})^2+1+27(\frac{\ell}{t})^4+30(\frac{\ell}{t})^2+3]$$

$$= \frac{p^2}{4}\,[39(\frac{\ell}{t})^4+26(\frac{\ell}{t})^2+4]$$

$$= p^2[3(\frac{\ell}{t})^2+1]^2 \qquad (56)$$

$$f\sigma_y = p[3(\frac{\ell}{t})^2+1] \qquad t = \frac{\ell}{\frac{1}{3}\sqrt{(\frac{f\sigma_y}{p}-1)}} \qquad (57)$$

Torque $T = 2p\ell w(\frac{D}{2} + \frac{\ell}{2}) = p\ell w(D+\ell) = pw(D\ell+\ell^2)$ (58)

Shaft torsional shear $\tau = \frac{16T}{\pi D^3}$ (59)

If the maximum torsional shear is taken as $\tau_{max} = .65\ f\sigma_y$, then

$$.65\ f\sigma_y = \frac{16T}{\pi D^3} \quad (60)$$

This defines minimum diameter $D_{min} = [\frac{16T}{\pi.65f\sigma_y}]^{1/3}$ (61)

In general, calculation of this minimum diameter serves only as a check because the diameter is usually dictated by other considerations. The most economical design of these actuators is to attach it to the already existing rudder stock. This is an example of the diameter being determined by other considerations.

Calculation of the thickness and stresses in the fixed vanes depends on the method of their attachment. If they are integral to the housing, then they have the same formulas as the moving vanes. However, they could be pegged to the end walls in which case they become beams. It is probably safe to say that they are about the same size as the movable vanes.

For cylinder walls and end plates, the same formulas apply as did for the piston and cylinder calculations in the linear actuator section.

Basing weights on the above formulas will be low by a considerable amount. This is because the required elaborate sealing system requires

$$\text{Torque } T = 2pLw\left(\frac{D}{2} + \frac{L}{2}\right) = pLw(D+L) = pw(DL+L^2) \qquad (52)$$

$$\text{Shaft torsional shear} \quad \tau = \frac{16T}{\pi d^3} \qquad (53)$$

If the maximum torsional shear is taken as $\tau_{max} = .65\, f_y$, then

$$\frac{16T}{\pi d^3} = .65\, f_y \qquad (54)$$

$$\text{This defines minimum diameter} \quad d_{min} = \left[\frac{16T}{\pi .65 f_y}\right]^{1/3} \qquad (55)$$

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For cylinder walls and end plates, the same formulas apply as they did for the piston and cylinder calculations in the linear actuator section.

Basing weights on the above formulas will be low by a considerable amount. This is because the required elaborate sealing system requires extensive space and weight. Therefore large fluctuations in weight can be expected depending upon the type of seals used and other empirical quantities such as manufacturing convenience and practices. However, in spite of the inaccuracies involved, the use of these proposed weight equations can be expected to show the trend of the weight with variations in the parameters. Weight of vane

$$W_v = \gamma_s \ell w t = \gamma_s \frac{\ell^2 w}{\sqrt{\frac{1}{3}\left(\frac{f\sigma_v}{p} - 1\right)}} \tag{62}$$

$$W_v = \frac{\gamma_s\left(\frac{T}{p} - D\ell w\right)}{\sqrt{\frac{1}{3}\left(\frac{f\sigma_y}{p} - 1\right)}} \tag{63}$$

3.2.2.2 <u>Weight Equations</u>

The equation for the overall weight of the rotary vane is

$$W = 4\gamma_s \ell w t + \gamma_s \pi (R_o^2 - R_i^2) w + 2\gamma_s \pi \left(R_o^2 - \frac{D^2}{4}\right) t_e$$

$$+ \gamma_s \pi \frac{D^2}{4} (w + 2t_e) + \gamma_o w \pi \left[R_i^2 - \frac{D^2}{4}\right] - 4t\ell \tag{64}$$

= 4 vanes + cylindrical shell + 2 ends + shaft + oil

$$W = 4\ell w t(\gamma_s - \gamma_o) + \pi w \frac{D^2}{4} (\gamma_s - \gamma_o) + \pi w R_i^2 (\gamma_o - \gamma_s)$$

$$+ \pi \gamma_s R_o^2 (wt2t_e) \tag{65}$$

but $\quad R_o = kR_i \quad$ and $\quad R_i = \frac{D}{2} + \ell$

$$R_o^2 = k^2R_i^2 \qquad R_i^2 = \frac{D^2}{4} + D\ell+\ell^2$$

$$W = 4\ell wt(\gamma_s-\gamma_o)+\pi w \frac{D^2}{4} (\gamma_s-\gamma_o)+\pi w(\frac{D^2}{4} + D\ell+\ell^2)(\gamma_o-\gamma_s)$$

$$+ \pi\gamma_s k^2(\frac{D^2}{4} + D\ell+\ell^2)(wt2t_e)$$

$$= w(\gamma_s-\gamma_o)[4\ell t + \frac{\pi D^2}{4} - \frac{\pi D^2}{4} - \pi D\ell-\pi\ell^2]$$

$$+ \pi\gamma_s k^2(\frac{D^2}{4} + D\ell+\ell^2)(w+2t_e)$$

$$W = (\gamma_s-\gamma_o)[4t\ell w-\pi \frac{T}{p}] + \pi\gamma_s k^2(\frac{D^2}{4} + \frac{T}{pw})(w+2t_e) \qquad (66)$$

Now substitute in for thicknesses (t = cylinder wall thickness, t_e = end thickness)

$$t = \frac{\ell}{\sqrt{\frac{1}{3}(\frac{f\sigma_y}{p} - 1}} \qquad (57)$$

$$t_e = R_o\sqrt{\frac{\alpha p}{f\sigma_y}} \qquad (15)$$

$$W = (\gamma_s-\gamma_o)[\frac{4\ell^2 w}{\sqrt{\frac{1}{3}(\frac{f\sigma_y}{p} - 1)}} -\pi \frac{T}{p}]+\pi\gamma_s k^2(\frac{D^2}{4} + \frac{T}{pw})[w+2k(\frac{D}{2}+\ell)\sqrt{\frac{\alpha p}{f\sigma_y}}]$$

(67)

where

$$k^2 = \frac{\frac{f^2\sigma_y^2}{p^2} + \sqrt{\frac{4f^2\sigma_y^2}{p^2} - 3}}{\frac{f^2\sigma_y^2}{p^2} - 3} \tag{21}$$

There are strong incentives to optimize with respect to length of sealed periphery since leakage rate is directly proportional to it.

$$\text{Periphery length} = 4(2\ell+w)+2\pi D = L_p \tag{68}$$

$$L_p = 8\ell+4w+2\pi D$$

but

$$w = \frac{T}{p} \frac{1}{(D\ell+\ell^2)} \tag{58}$$

$$L_p = 8\ell + \frac{4T}{p\ell(D+\ell)} + 2\pi D \tag{69}$$

Since ℓ and D are independent, the following procedure may be used to optimize periphery[17]

$$\frac{\partial L_p}{\partial \ell} = 8 + \frac{4T}{p} \frac{(-)(D+2\ell)}{(D\ell+\ell^2)^2} = 0 \tag{70}$$

$$\frac{D+2\ell}{D^2\ell^2+2D\ell^3+\ell^4} = \frac{2p}{T} \tag{71}$$

$$\frac{\partial L_p}{\partial D} = \frac{4T}{p} \frac{(-)(\ell)}{(D+\ell^2)^2} + 2\pi = 0 \tag{72}$$

$$\frac{1}{\ell(D+\ell)^2} = \frac{p}{2T} \tag{73}$$

$$\sqrt{\frac{2T}{p\pi\ell}} = D+\ell \qquad D = \sqrt{\frac{2T}{p\pi\ell}} - \ell \tag{74}$$

From equation (70)

$$D+2\ell = \frac{2p}{T}\,(D\ell+\ell^2)^2 \tag{75}$$

Now substitute to eliminate D

$$\sqrt{\frac{2T}{p\pi\ell}} -\ell+2\ell = \frac{2p}{T}\;\frac{2T\ell}{\pi p} = \frac{4\ell}{\pi}$$

$$\sqrt{\frac{2T}{p\pi\ell}} = \frac{4\ell}{\pi} -\ell = \ell(\frac{4}{\pi} - 1)$$

$$\frac{2T}{p\pi\ell} = \ell^3(\frac{4}{\pi} - 1)^2 \tag{76}$$

Then the optimum ℓ to minimize periphery is determined to be

$$\ell_{OPT} = [\frac{2T}{p\pi(\frac{4}{\pi} - 1)^2}]^{1/3} \tag{77}$$

Now solve for optimum D.

$$D_{OPT} = \sqrt{\frac{2T}{p\pi[\frac{2T}{p(\frac{4}{\pi} - 1)^2}]^{1/3}}} - [\frac{2T}{p\pi}\;\frac{1}{(\frac{4}{\pi} - 1)^2}]^{1/3} \tag{78}$$

$$= \sqrt{\left(\frac{2T}{p\pi}\right)^{2/3}\left(\frac{4}{\pi} - 1\right)^{2/3}} - \left[\frac{2T}{p\pi}\ \frac{1}{\left(\frac{4}{\pi} - 1\right)^{2}}\right]^{1/3}$$

$$= \left[\frac{2T}{p\pi}\left(\frac{4}{\pi} - 1\right)\right]^{1/3} - \left[\frac{2T}{p\pi}\ \frac{1}{\left(\frac{4}{\pi} - 1\right)^{2}}\right]^{1/3}$$

$$= \left(\frac{2T}{p\pi}\right)^{1/3}\left[\left(\frac{4}{\pi} - 1\right)^{1/3} - \frac{1}{\left(\frac{4}{\pi} - 1\right)^{2/3}}\right]$$

$$D_{OPT} = \left(\frac{2T}{p\pi}\right)^{1/3} \frac{\frac{4}{\pi} - 2}{\left(\frac{4}{\pi} - 1\right)^{2/3}} = \text{(negative)} \qquad (79)$$

But D cannot be negative and therefore the minimum is not physically realizable. The calculation does serve a useful purpose in indicating that the minimum D possible from other considerations will give minimum leakage.

Notice that the periphery decreases with pressure because the size of the vanes required to produce the given torque decreases. Thus increasing pressure does produce some small effects which help to reduce leakage rate. Notice also that the equation for optimum ℓ does not include D among its variables. Therefore this may be used as an independent equation to replace ℓ in the overall weight equation. The equation for D cannot be used for this purpose since it is physically unobtainable. D must remain in the weight equation as a parameter determined by the requirements of the application.

The weight equation can now be written in terms of the independent variables w, p, $f\sigma_y$, and D. Because the variables are independent, an

optimum value of w, in terms of the remaining variables, can be determined from the equation $\frac{\partial W}{\partial w} = 0$.

$$(\gamma_s-\gamma_o)\frac{4\ell^2}{\sqrt{\frac{1}{3}\left(\frac{f\sigma_y}{p}-1\right)}} + \pi\gamma_s k^2\frac{D^2}{4} - \frac{\pi\gamma_s k^2 T_2 k}{pw^2}\left(\frac{D}{2}+\ell\right)\sqrt{\frac{\alpha p}{f\sigma_y}} = 0 \tag{80}$$

$$w_{OPT} = \left\{\left[\frac{(\gamma_s-\gamma_o)4\ell^2}{\sqrt{\frac{1}{3}\left(\frac{f\sigma_y}{p}-1\right)}} + \frac{\pi\gamma_s k^2 D^2}{4}\right]\left[\frac{p}{2\pi\gamma_s k^3 T\left(\frac{D}{2}+\ell\right)\sqrt{\frac{\alpha p}{f\sigma_y}}}\right]\right\}^{1/2} \tag{81}$$

This value may be substituted back into the weight equation which is now a variable of only p, $f\sigma_y$, and D. This can be optimized with respect to $p/f\sigma_y$. That is the value of $p/f\sigma_y$ can be determined which will give the minimum weight of the rotary vane actuator that has values of ℓ optimized to give minimum leakage rates. Unfortunately, the equation is too unwieldy to be manipulated to determine this optimum value of $p/f\sigma_y$ for values of D as was done in the case of the piston and cylinder.

3.2.2.3 Problem Definition

At this point, it is important to define the problem in terms of the variables. The weight of the system can be divided into three parts. They are (a) the rotary vane weight, (b) the pump weight, and (c) the electric drive motor weight. The weight of the rotary vane has been

shown to be reduceable to a function of p, $f\sigma_y$, and D. Now consider pump weight. As has been shown in the section on linear actuators, the best procedure is to select a pump that is applicable to the range of interest and consider pump weight as constant in this range. However, in order to determine the range of interest, the volume flow rate must be calculated. This depends strongly on the leakage rate which can only be determined by knowing the pressure. The motor weight is a direct function of horsepower. The power requirements can be divided into two parts. The first is the power used in turning the rudder and the second is the leakage power.

$$p_{motor} = C_1 \frac{\text{(torque)(rudder rate)}}{\text{efficiency}} + C_2 p^e \tag{82}$$

where the exponent e has been shown to be about 1.3 at higher pressures.

It is clear from the above definition of the problem that an analytic approach to its solution is not possible. If it is recognized that for a steering engine, the value of D for the rotary vane is usually determined by consideration of factors not involving optimization, another approach to the problem suggests itself. This method is to assume that D and p are fixed and constant and then determine the optimum ℓ and w to minimize leakage for this combination. The definition of the problem is now to minimize L_p subject to the constraints shown to [illegible]

pump weight. [illegible]

the heat [illegible]

of interest and [illegible]

ever, in order [illegible]

rate must be calculated. [illegible]

which can [illegible]

is a direct function [illegible]

divided into two parts. [illegible]

rudder and [illegible]

[illegible]

It is clear from [illegible] approach to its [illegible] steering engine, [illegible] by consideration [illegible] to the problem [illegible] are fixed and [illegible] leakage for this consideration. [illegible] minimize L_p subject to [illegible]

(A) $T = p\, w(D+\ell) = \text{constant}$

(B) $p = \text{constant}$

(C) $D = \text{constant}$

Then the effect of p can be determined by calculations at several different p's. Notice that ℓ and w are no longer independent. Constraint (A) can be substituted into the equation for periphery to give

$$L_p = 8\ell + \frac{4T}{p(D\ell+\ell^2)} + 2\pi D \tag{83}$$

The optimum ℓ is now found by setting the differential to zero.

$$\frac{dL_p}{d\ell} = 0 = 8 - \frac{4T}{p} \frac{(D+2\ell)}{(D\ell+\ell^2)^2} \tag{84}$$

$$\frac{D+2\ell}{(D+\ell)^2} = \frac{2p}{T} \ell^2 \tag{85}$$

$$\ell^4+2D\ell^3+D^2\ell^2 - \frac{T}{p} \ell - \frac{TD}{2p} = 0 \tag{86}$$

This equation defines the optimum ℓ. A similar equation can be found for the optimum w. Proceed by solving equation (85) for ℓ in terms of w.

$$(D+2\ell) \frac{1}{(D+\ell)^2} = (D+2\ell) \frac{p^2\ell^2w^2}{T^2} = \frac{2p\ell^2}{T} \tag{87}$$

$$\ell = \frac{T}{pw^2} - \frac{D}{2} \qquad (88)$$

This is now substituted into the constraint equation (A)

$$T = pw\left(\frac{T}{pw^2} - \frac{D}{2}\right)\left(D + \frac{T}{pw^2} - \frac{D}{2}\right) \qquad (89)$$

$$T = -\frac{pwD^2}{4} + \frac{T^2}{pw^3} \qquad (90)$$

$$w^4 + \frac{4T}{pD^2}\,w^3 - \frac{4T^2}{p^2D^2} = 0 \qquad (91)$$

This equation defines the optimum w to minimize periphery. Unfortunately the expressions for the optimums of both w and ℓ are quartic equations and an analytic solution is too unwieldy to warrant the effort. Without solving them directly, use of Descarter Rule of Signs indicates that there is only one positive real root for each equation. Therefore the optimum values are physically realizable. The design procedure that is suggested is to solve for optimum w or ℓ, by trial and error, determine the other one from constraint equation (A) and use these to calculate weights.

3.2.2.4 Design Calculation

Calculate design values for D = 24", p = 3000 and 5000 psi, and $f\sigma_y$ = 17500 as before.

5000 psi, w = 7.78", ℓ = 4.5", from equations 86 and 91

This is [illegible]

This [illegible] tunately the [illegible] equations [illegible] the effect Without solv[illegible] that there is [illegible] the optimum value [illegible] is suggested [illegible] the other one [illegible] weights.

3.2.2.4 Design [illegible]

Calculate [illegible]

$f\sigma_y$ = 17500 as before

5000 psi, [illegible]

$$t = \frac{\ell}{\sqrt{\frac{1}{3}(\frac{f\sigma_y}{p} - 1)}} = \frac{4.5}{\sqrt{\frac{2.5}{3}}} = 4.92''$$

$$k^2 = \frac{\frac{f^2\sigma_y^2}{p^2} + \sqrt{\frac{4f^2\sigma_y^2}{p^2} - 3}}{\frac{f^2\sigma_y^2}{p^2} - 3} = \frac{12.25 + \sqrt{49-3}}{12.25-3} = 2.06$$

$$k = 1.4$$

inner diameter = 33.0" , outer diameter = 46.2"

$$t_e = \sqrt{\frac{(.74)(5000)(13.5^2)}{17500}} = 7.58''$$

Weights	cylinder	$= (.283)(\pi)(\frac{33.0^2}{4})(2.06-1)(7.78)$	= 1,912
	vanes	$= 4(.283)(7.78)(4.5)(4.92)$	= 195
	ends	$= 2(.283)(\pi)(23.2^2-12^2)(7.58)$	= 5,300
	oil	$= (.033)(7.78)[\pi(\frac{33.0^2}{4} - 144)-4(4.92)(4.5)]$	= 72
		TOTAL	7,479 lbs

<u>3000 psi</u> $w = 9.7$ $\ell = 5.8$

$$t = \frac{5.8}{\sqrt{\frac{4.85}{3}}} = 4.56''$$

$$k^2 = \frac{34.2 + \sqrt{136.8-3}}{34.2-3} = \frac{34.2+11.56}{31.2} = 1.467$$

$k = 1.211$

inner diameter = 35.6 , outer diameter = 43.1

$$t_e = \sqrt{\frac{(.74)(3000)(17.8^2)}{17500}} = 6.34$$

Weights

cylinder	$= (.283)(\pi)(\frac{35.6^2}{4})(1.467-1)(9.7)$	=	1,275
vanes	$= 4(.283)(9.7)(5.8)(4.56)$	=	290
ends	$= 2(.283)(\pi)(21.55^2-12^2)(9.7)$	=	5,520
oil	$= (.033)(9.7)[\pi(\frac{35.6^2}{4} - 144)-4(4.56)(5.8)]$	=	140
			7,225 lbs

Now estimate the size and weights of the pumps and drive motors. The mechanical efficiency of a rotary vane is generally about 95%. First calculate the flow rate required to produce the output horsepower. Then estimate the leakage rate. The sum of these is the required pumping rate which then determines the size of the pump and the drive horsepower.

3000 psi

$$\frac{31\ HP_{rudder}}{.95} = \frac{3000\ lbs}{in^2}\ \frac{144\ in^2}{ft^2}\ Q\ \frac{gal}{min}\ \frac{ft^3}{7.481\ gal.}\ \frac{HP\ min}{33000\ ft\ lbs}$$

$$Q = \frac{(31)(7.481)(33000)}{(.95)(3000)(144)} = 18.68\ gpm$$

$k = 1.211$

inner diameter = 35.[illegible] outer diameter = 43.[illegible]

$$t_e = \sqrt{\frac{(.74)(3[illegible])(17.[illegible])}{17500}} = [illegible]$$

Weights

cylinder = $(.283)(4)(\ldots)(1.457-1)(9.7)$ = [illegible]

vanes = $4(.283)(\ldots)(\ldots)(\ldots)$ = [illegible]

ends = $2(.283)(4)(\ldots)(\ldots)$ = [illegible]

oil = [illegible]

Now estimate the size and weights of the pumps and drive motors. The mechanical efficiency of a rotary vane is generally about 95%. First calculate the flow rate required to produce the output horsepower. Then estimate the leakage rate. The sum of these is the required pumping rate which then determines the size of the pump and the drive horsepower.

<u>3000 psi</u>

$$\frac{31\ HP_{rudder}}{.95} = \frac{3000\ lbs}{in^2}\ \frac{144\ in^2}{ft^2}\ \frac{gal}{min}\ \frac{ft^3}{7.481\ gal}\ \frac{min}{33000\ ft\ lbs}$$

$$Q = \frac{(31)(7.481)(33000)}{(.95)(3000)(144)} = 1[illegible].5\ gpm$$

Estimate leakage from Fig. (XIII)

Required pumping rate = 20.40 gpm

Hydreco pump requires 40.1 HP input. Therefore a 30 HP electric motor is required.

Weights	Hydraulic pump		110 lbs
	Motor		410 lbs
		TOTAL	520 lbs

5000 psi

$$Q = \frac{(31)(7.481)(33000)}{.95\ (5000)\ (144)} = 11.20 \text{ gpm}$$

Estimate leakage from Fig. (XIII)

Required pumping rate = 12.99 gpm

Hydreco pump requires 48 HP input. Therefore a 40 HP electric motor is required.

Weights	Hydraulic pump		110 lbs
	Motor		492 lbs
		TOTAL	602 lbs

3.2.2.5 Weight Summary

	3000 psi	5000 psi
cylinder	1,275	1912
vanes	290	195
ends	5,520	5,300
oil	140	72
hydraulic pump	110	110

A.C. drive motor	410	492
TOTAL	7,745 lbs	8,081 lbs

3.2.2.6 Discussion

Perusal of the foregoing design and weight calculations turns up several interesting points. The first among these concerns the leakage rate. Certainly the method of estimating leakage from Fig. (XIII) lacks a great deal in preciseness. However, it does indicate a reasonable order of magnitude for what the leakage rate might be expected to be. The significant point about leakage is how little relative effect it has on the overall design. Even if it were doubled, it could still be handled by the same pump with the same motor already selected in each case. As expected, leakage rate increases with pressure and at 5000 psi it is nearly 14% of the pumping rate. Although this does force the selection of a larger motor,than the one for 3000 psi, the increase in weight of eighty pounds is only one percent of the total weight. This suggests that although leakage rate is an area requiring careful design attention, it is not a significant factor in determining the weight and size of a rotary vane steering engine.

The great majority of the weight of the system is accounted for by the cylindrical shell and the end plates. This is because the thicknesses work out to be a given percentage of the diameter for a given pressure. Therefore the thickness will be large for the larger diameters inherent in a rotary vane. The heavy concentration of weights in these two

components suggests that attempts at weight reduction be focused primarily on them. In fact, it appears that a valid initial simplification to the problem would be to neglect the weights of the vanes and the oil. Suggested ways of reducing the weight would be to elongate the unit to reduce diameter, and to use higher yield strength steel to reduce the thicknesses. The above simplifications make the problem very similar to the piston and cylinder analysis performed in the linear actuator section.

A set of curves very similar to Fig. (VI) can be drawn by relatively modest changes to the pertinent equations. As a matter of fact, some insight into the rotary vane can be gained by examining Fig. (VI). It can be expected that there will be an optimum value of $\frac{p}{f\sigma_y}$. The effects of increasing ℓ cannot be taken from this graph because equation (58) does not apply to a piston and cylinder. This equation indicates that diameter will decrease as ℓ is decreased which will make the shell thinner. This causes weight to decrease. However, as ℓ is decreased axial length increases causing a weight rise. An optimum can be located where these two effects balance. It appears from this that the lightest arrangement is likely to be a relatively long arrangement. As a matter of fact, this type of design has already evolved abroad.[21] If this proposed analysis were performed, substantial reduction in the weights listed above could be expected.

The final point of interest growing out of the design study is the relatively small size of the vanes. The span and thickness of the vanes would decrease further if the axial length of the unit were increased for

the reasons just expressed. These small dimensions suggest the possibility of manufacturing them as an integral part of the rudder stock by machining them directly out of the same billet. No attempt is made here to evaluate the feasibility of this proposal. It is mentioned only to point out that it is desirable from the point of view of weight and design simplicity.

In summary, it is clear that the rotary vane is feasible for this torque range and no technical reason is evident that would exclude its application to any other torque range. Pressures up to 5000 psi are feasible, but their desirability must be dependent on calculation of an optimum $p/f\sigma_y$ value. Increasing axial length of the unit is desirable. The system weights which are given above can probably be reduced substantially should the suggested optimization procedure be carried out.

3.3 ELECTRO-MECHANICAL DEVICES

3.3.1 Gear Reduction Drives

3.3.1.0 Gear Trains

Historically, the geared quadrant has seen extensive application in steering engines[22]. It was rugged, simple and reasonably efficient, but it has largely disappeared from use today. In order to obtain high reduction ratios and reduce the tooth loading on and the size of the drive gears, large quandrant radii were used. This made the system so large and cumbersome that it was superseded by the electro-hydraulic machines.

The worm gear offers a convenient light weight design easily capable of the reduction ratios required for this application. Unfortunately it suffers from the two defects of low efficiency and large dimensions. In order to reduce tooth loading, the radius of the quadrant must be increased to both lower the tangential force and increase the arc of contact with the worm. The low efficiency, usually less than fifty percent, is caused by the large sliding contact area and requires the installation of a much larger drive motor. For these reasons the worm drive has not found wide acceptance.

Another type of gear system capable of achieving the high reduction ratios required of steering engines is the epicyclic family of gear trains. There are several sub groups including simple epicyclic, compound epicyclic, differential, and fixed differential systems each with a myriad of possible arrangements. They may be coupled or used in conjunction with other gear types. These systems can have high capacities because the number of planet gears may be increased providing more contact area to carry the load. For high speed applications, this family of gear trains is capable of providing very high reduction ratios in an

extremely compact package. However with the high torques and low speeds involved in a steering engine, the size of the gear becomes larger and these systems lose their attractiveness. In addition, there are other specific problems. The simple and compound epicyclic gear trains have lower possible reduction ratios which requires that several of them be connected in series to produce the desired reduction. Then the assembly becomes complicated, and it is difficult to provide adequate support for the bearings, particularly for the planets, which have very high cantilever loadings. It is also difficult to distribute the loading equally among all the gears, and alignment, balance and vibration may also become problem areas. The fixed differential is capable of very high reduction ratios (up to 500:1), but of course with the same difficulties attendent to the simple and compound epicyclic trains. Although it is the lightest of the epicyclic types its efficiency may be quite low (26%) because it has a large amount of tooth meshing occurring at high tooth loads[23]. In summary an epicyclic gear train would have several problems associated with it which could be minimized only with very extensive and clever design work. It would be complicated and have a large number of moving parts. Because of the high torque involved here, it would probably be heavier than the presently existing systems.

<u>3.3.1.1 High Ratio Mechanisms</u>

In the past, several arrangements of a screw and linkage have been used in steering engines, the Napier screw being one common example[22]. Although this design may be made more compact than the quadrant type gear, it has an inherently low efficiency. As with the worm gear, the screw has a large amount of surface area in sliding contact under heavy load. For these reasons, its usage has generally declined.

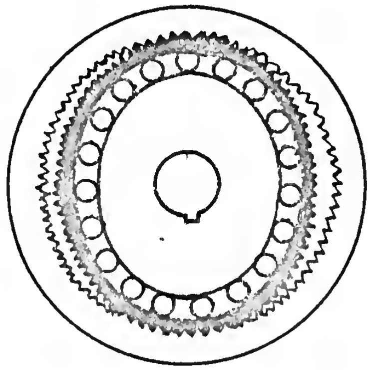
Another type of high ratio device is the harmonic gear shown in Figure (XX). This is a relatively new invention in which a rotating cam distorts a thin toothed cylinder into contact with a rigid toothed member. There is a small difference between the numbers of teeth on the flexible and rigid members so that for every rotation of the cam, these members rotate only a few tooth widths with respect to each other. The cam is called the wave guide, the flexible member the flexspline and the rigid member the circular spline. One member (any one) must be fixed and the system can be arranged so that the input and output may be taken from any member. The device is capable of reduction ratios of up to 350:1 and may be configured to meet a wide variety of applications. The spline teeth come into contact with almost pure radial motion and therefore have extremely low sliding velocities. This results in low tooth wear and low friction losses which give the gear very high efficiencies (86% for 100:1 ratio). It has two tooth areas engaging up to 10% of the pitch diameter which gives unusually high load capacity. The components are tubular which is a particularly efficient form for torque transmission, and it allows concentric arrangement of the parts. All of these factors taken cumulatively result in making the harmonic gear the lightest and most compact of all the high reduction ratio, high torque capacity, high efficiency gear systems[23].

Another type of high reduction ratio gear whose weight and efficiency are close to those of the harmonic gear is the planocentric or cycloidal cam driven gear shown in Figure (XX). In this design the input shaft causes an internal pinion gear to wobble and in so doing walk around the inside of the periphery of a fixed annulus gear. The high reduction is due to the small difference between the number of teeth in the pinion and

FIGURE XX

HARMONIC GEAR AND PLANOCENTRIC GEAR

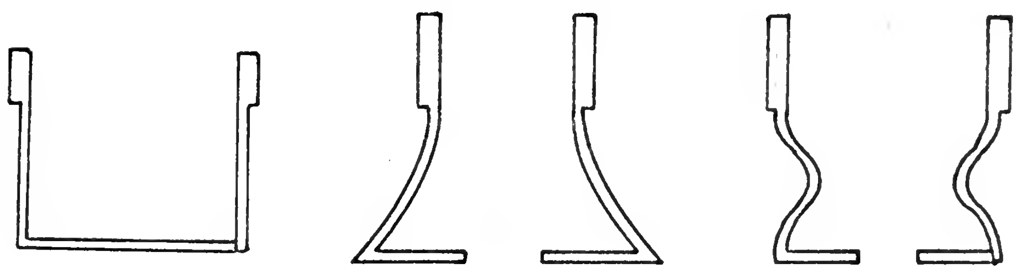
Top view



wave generator
ball bearing rod
flexspline
circular spline

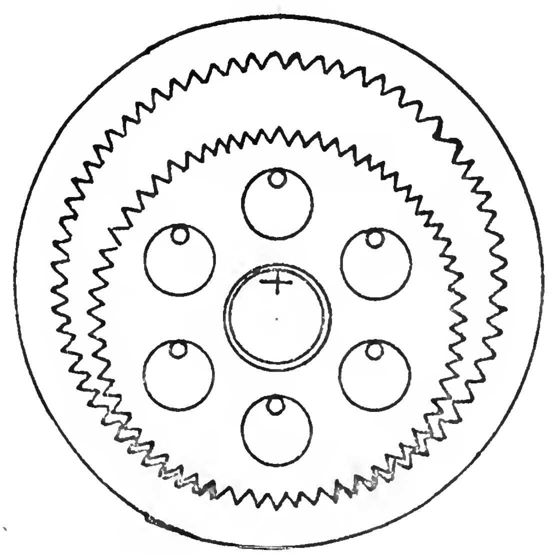
Harmonic Gear

Flexspline Configurations



cup　　bell　　inverted bell

Drive end view



fixed to gear case
wobbling pinion
drive shaft axis
drive cam
pins connected to output shaft

PLANOCENTRIC GEAR

annulus gears. Efficiencies as high as 90% are attainable due to the spur gear tooth form. However, the gear is essentially an unbalanced design and has inordinately high bearing loading[23]. It only has one area of tooth contact and for this reason it cannot be expected to have as high a torque capacity per unit face width as the harmonic gear.

When all of the gear reduction drives described above are compared, the harmonic gear emerges as the most promising device. It was therefore selected for the detailed analysis that follows.

3.3.2 Harmonic Gear

3.3.2.0 General

There are several basic methods of elastically deflecting the flexspline. Consider first the hydraulic piston actuator. In this design a series of hydraulic pistons are located radially around the flexspline. Then pressurized oil is fed to them by a centrally located rotating valve, causing the pistons to produce the eliptical deflection required in the flexspline[24]. This system offers the possibility of driving a 200:1 reduction ratio harmonic gear with a high speed squirrel cage motor and hydraulic pump with no intermediate transmission. Unfortunately, it would consume substantial power even during the time that the rudder was not being turned. Of more serious consequence is the characteristic of this design that if there were even a momentary loss of power, the flexspline would be disengaged from the circular spline allowing the rudder to swing free. The only way to prevent this would be to install some sort of brake on the rudder stock which would be actuated in the event of a power loss. However, it would have to have a capacity of the full five million inch pounds, and the dimensions of such a brake are stupendous.

Another design utilizes direct electromagnetic action on the flex-

spline. A series of electrical solenoid coils are arranged radially around the periphery of the flexspline. Then by programming the correct amount of current to each coil, the flexspline deflection shape is achieved by solenoid action[25]. This is inherently a low torque device which also suffers from the same disadvantages as the pressure actuated design, and hence it is not adaptable to a steering engine application. Its most attractive characteristic is that it has one of the lowest inertias of all rotational actuators, and the exceptionally high response speed resulting from this may well find application in the aerospace field.

The standard design uses ball bearings around an eliptical cam to achieve the required deflection of the flexspline. For ℓ/D values greater than one fifth, several rows of ball bearings are used. Although roller bearings would have a higher load capacity, a great deal of experimental work would have to be done to determine the proper taper of the rollers to achieve a good load distribution. In view of the satisfactory performance of the ball bearings, such work does not appear justified[26].

Frequently the lifetime of the harmonic gear is dependent on the lifetime of the ball bearings. In those applications which are restricted by these lifetimes, the use of a hydrostatic bearing appears to offer a solution. A thin film of oil at high pressure would be pumped into a clearance space between the eliptical cam and the flexspline. Since the clearance space is only of the order of several thousandths of an inch, loss of oil pressure would not result in disengagement of the teeth. The source of oil is readily available if a hydraulic drive is used, and the steady-state power loss is reasonably small. In addition to entirely

removing lifetime considerations from harmonic gear design, it would improve the already high efficiency under operation. Unfortunately, no developmental work has been done on such a hydrostatic bearing, and a design for it does not presently exist[26]. Therefore, its use will not be considered in this paper. However, it is one of the avenues open to future development of the harmonic gear and its application as a steering engine.

The general approach to the analysis of the harmonic gear will be to start with, a basic analysis of the stresses in the components of the gear. The equations used will yield only a crude approximation to the stresses actually existent. However they do give a valid insight into the parameters involved in calculating dimensions and weights for the harmonic gear. With these parameters in mind, an appropriate arrangement of the gear will be developed which adapts the harmonic gear to best fit the requirements of a ships steering engine. Weights of this arrangement will be calculated. Then a control analysis will be performed which will determine the response characteristics and control system requirements for this arrangement. Finally, general observations on the compatibility of the harmonic gear with the overall ship system will be made.

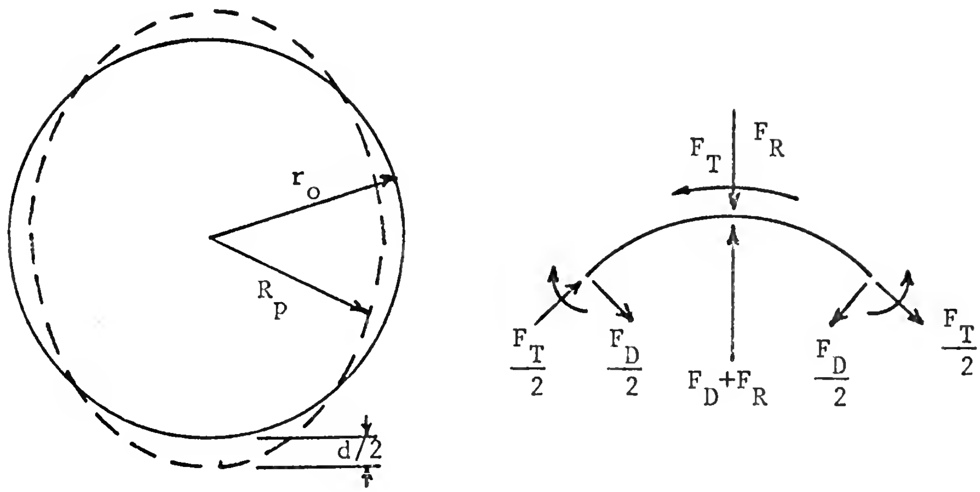
3.3.2.1 Stress Analysis

The stress analysis will be done for the standard harmonic gear configuration which has the wave generator located inside the flexspline and the circular spline outside of it. The analysis and its conclusions are equally applicable to any of the other possible configurations of the components. Calculation of the deflection forces and bending moments is in general based on energy methods for a thin ring model.

First consider the flexspline. Bending moments are worked out and

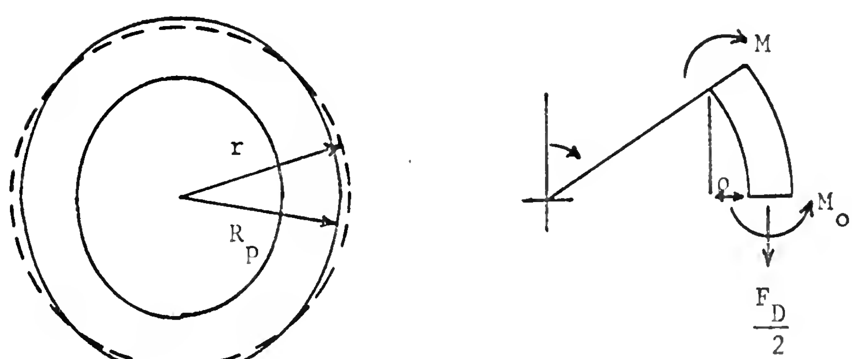
FIGURE XXI

NOMENCLATURE FOR HARMONIC GEAR COMPONENTS

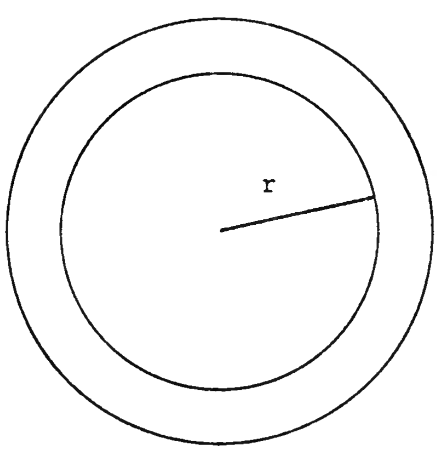


Flexspline

Wave Generator



Circular Spline



ℓ = face width of gears, i.e., the length of the teeth in the axial direction.

D = pitch diameter = diameter of flexspline.

then the deflection force is arrived at by use of the theorem of virtual work. Then a calculation of the bending stresses is worked out. Then a very rough approximation is worked out for the relationship between stress and torque capacity.

$$R_p = r_o + \frac{d}{2}\cos 2\theta \tag{92}$$

$$u = \frac{d}{2}\cos 2\theta, \quad u'' = -2d\cos 2\theta \tag{93}$$

$$M = EI\,\frac{u+u''}{r_o^2} = EI\,\frac{(\frac{d}{2}\cos 2\theta - 2d\cos 2\theta)}{r_o^2} \quad \text{equation 122a of reference [11]} \tag{94}$$

Take convention that bending moment which tends to decrease curvature is (+). Then at $\theta = 0$, this M is (-)

$$M = -EI\,\frac{3d\cos 2\theta}{2r_o^2} \tag{95}$$

$$U = \frac{1}{2EI}\int M^2\, ds \tag{96}$$

$$= \frac{1}{2EI}\int_o^{2\pi} \frac{E^2I^2\, 9d^2\cos^2 2\theta}{4r_o^4}\; r d\theta$$

$$U = \frac{9d^2\, EI\pi}{8r_o^3} \tag{97}$$

$$\frac{\partial u}{\partial d} = \frac{9\,EI\pi}{48r_o^3}\,2d = \frac{9\pi\, dEI}{4r_o^3} = F_D \text{ by theorem of virtual work} \tag{98}$$

$$F_D = \frac{7.07dEI}{r_o^3} \qquad \text{if } I = \frac{\ell t^3}{12} \tag{99}$$

$$F_D = \frac{.59dE\,t^3}{r_o^3} \quad \text{but } r_o = \frac{D-d}{2} \tag{100}$$

$$F_D = \frac{4.72eE\ell t^3}{(D-d)^3} = \frac{4.72eE\ell t^3}{d^3-3D^2d+3d^2D-d^3} \tag{101}$$

$$F_D = \frac{4.72dE\ell t^3}{D^3(1-\frac{3}{R_D}+\frac{3}{R_D^2}-\frac{1}{R_D^3})} \quad \text{where } R_D = (-)\frac{D}{d} \tag{102}$$

= reduction ratio of gear

In order to determine the bending stresses use the approximate equation on page 226 of reference [11].

$$\sigma_B = \frac{12M}{bh^3}\left(z + \frac{h^2}{12R_o} - \frac{z^2}{R_o}\right) \quad \text{where } h = t = \text{thickness} \tag{103}$$

z = 1/2 thickness

$b = \ell$ = face width of gear

R_o = mean radius

Now substitute the following relations

$$M = -EI\,\frac{3}{2}\,\frac{d\cos 2\theta}{r_o^2} \qquad I = \frac{\ell t^3}{12} \tag{104}$$

$$M = -\frac{E\,t^3}{12}\,\frac{3}{2}\,\frac{d\cos 2\theta}{r_o^2} \tag{105}$$

$$\sigma_B = \frac{12}{\ell t^3}\,\frac{E\ell t^3}{12}\,\frac{3}{2}\,\frac{d\cos 2}{r_o^2}\left(\frac{t}{2} + \frac{t^2}{12R_o} - \frac{t^2}{4R_o}\right) \tag{106}$$

$$r_o = \frac{D\left(1 - \frac{1}{R_D}\right)}{2}, \quad R_o = \frac{d}{2} \tag{107}$$

$$\sigma_B = \frac{3Ed\cos 2\theta}{2\,\frac{D^2}{4}\left(1 - \frac{1}{R_D}\right)^2}\left(\frac{t}{2} + \frac{t^2}{6D} - \frac{t^2}{2D}\right) \tag{108}$$

for $\quad \theta = .05\pi,\ \cos 2\theta \approx 1$

$$\sigma_B = \frac{3Edt}{D^2\left(1 - \frac{1}{R_D}\right)^2} - \frac{Edt^2\ 2}{D^3\left(1 - \frac{1}{R_D}\right)^2} \quad \text{(tension)} \tag{109}$$

If t is $\approx .02D$ and if $R_D > 100$, then have

$$\sigma_B \approx \frac{3Edt}{D^2} \quad \text{(tension in outer fiber)} \tag{110}$$

Now relate the stresses to the torque capacity T

$$\sum M = 0 = (2F - F_t)D \tag{111}$$

$$F = \frac{F_t}{2}$$

At right-hand side σ_T tensile stress due to F_t is

$$\sigma_t = \frac{\frac{F_t}{2}}{A} = \frac{T}{2D\ell t} \tag{112}$$

Then total tension in outer fiber

$$\sigma_t = \sigma_t + \sigma_B = \frac{T}{2D\ell t} + \frac{3Edt}{D^2} \tag{113}$$

then $\qquad t = D-D_i = D(1-k_F) \qquad D_i = k_F D$

$\qquad d = D/R_D \qquad R_D$ = reduction ratio

$$\sigma_T = \frac{T}{2D\ell D(1-k_F)} + \frac{3ED/R_D D(1-k_F)}{D^2} \tag{114}$$

$$\sigma_T = \frac{T}{2\ell(1-k_F)D^2} + \frac{3E(1-k_F)}{R_D} \tag{115}$$

$$T = (\sigma_T - \frac{3E(1-k_F)}{R_D})2\ell(1-k_F)D^2 \tag{116}$$

$$T = [2(1-k_F)\sigma_T - \frac{6E(1-k_F)^2}{R_D}]\ell D^2 \tag{117}$$

Note that the shear caused by $\frac{F_D}{2}$ is not considered. The important point here is that the expression for the torque capacity of the flexspline is of the form

$$T = (\text{Constant})\ell D^2$$

which can be written as

$$T = (\text{Constant})D^3 \left(\frac{\ell}{D} \right) \tag{118}$$

Calculation of the deflection force of the wave generator is done next using the same model. This is a much worse approximation than for the flexspline because it is not "thin" as assumed. The calculation leads to a relationship between D, the pitch diameter, ℓ the face width of the gear, and T the torque capacity. The fact that the wave generator is eliptical in shape causes the calculation to be somewhat

lengthy. The theorem of Castigliano is used to find the moment in the ring in terms of the applied force P. Then the theorem is used again to find the deflection δ produced by P.

$$R_p = r + \frac{d}{2} \cos 2\theta \tag{119}$$

$$\rho = R_{p_{\pi/2}} - R_{p_\theta} \sin\theta \tag{120}$$

$$\rho = (r + \frac{d}{2} \cos 2 \frac{\pi}{2}) - (r + \frac{d}{2} \cos 2\theta) \sin\theta \tag{121}$$

$$\rho = r - \frac{d}{2} - r \sin\theta - \frac{d}{2} \cos 2\theta \sin\theta \tag{122}$$

$$\rho = r(1 - \sin\theta) - \frac{d}{2}(1 + \cos 2\theta \sin\theta) \tag{123}$$

$$M = M_o = \frac{P}{2} \rho = M_o - \frac{P}{2} [r(1 - \sin\theta) - \frac{d}{2}(1 + \cos 2\theta\sin\theta)] \tag{124}$$

$$U = \frac{1}{2EI} \int M^2 \, ds \tag{125}$$

$$= \frac{1}{2EI} \int_o^{\pi/2} \{M_o - \frac{P}{2}[r(1-\sin\theta) - \frac{d}{2}(1 + \cos 2\theta\sin\theta)]\}^2 r d\theta \tag{126}$$

The term in the brackets is expended as follows

$$M_o^2 - P M_o[r(1-\sin\theta) - \frac{d}{2}(1 + \cos 2\theta\sin\theta)] + \frac{P^2}{4}[r^2(1-\sin\theta)^2$$
$$- rd(1-\sin\theta)(1 + \cos 2\theta\sin\theta) + \frac{d^2}{4} (1 + \cos 2\theta\sin\theta)^2] \tag{127}$$

The above expression can be simplified. Then multiply by r and integrate with respect to θ. After inserting the limits of integration the expression becomes

$$U = \frac{1}{2EI} \{M_o^2 r \frac{\pi}{2} - P M_o r[r(\frac{\pi}{2} - 1) - \frac{d}{2}(\frac{\pi}{2} + \frac{2}{3})]$$
$$+ \frac{P^2}{4} [(\frac{\pi}{2} - 2 + \frac{\pi}{4})r^3 - r^2 d(\frac{\pi}{2} + \frac{2}{3} - 1 + \frac{\pi}{8})$$
$$+ \frac{d^2 r}{r}(\frac{\pi}{2} + \frac{4}{3} + \frac{\pi}{8})]\} \tag{128}$$

then $\frac{\partial U}{\partial M_o}$ = 0 by the theorem of Castigliano because no angular rotation at midpoint (point of application of P).

$$\frac{\partial U}{\partial M_o} = 0 = \frac{4}{2EI}\left\{2r\,M_o\,\frac{\pi}{2} - P\,r\left[\left(\frac{\pi}{2}-1\right)r - \frac{d}{2}\left(\frac{\pi}{2}+\frac{2}{3}\right)\right] + 0\right\} \tag{129}$$

$$M_o = \frac{Pr}{\pi r}\left[r\left(\frac{\pi}{2}-1\right) - \frac{d}{2}\left(\frac{\pi}{2}+\frac{2}{3}\right)\right] \tag{130}$$

Substitute back to find M

$$M = \frac{P}{2}\left[r\left(1-\frac{2}{\pi}\right) - d\left(\frac{1}{2}+\frac{2}{3\pi}\right)\right] - \frac{P}{2}\left[r(1-\sin\theta) - \frac{d}{2}(1+\cos 2\theta\sin\theta)\right] \tag{131}$$

$$= \frac{P}{2}\left[r\left(\sin\theta - \frac{2}{\pi}\right) - \frac{d}{2}\left(\frac{4}{3\pi} - \cos 2\theta\,\sin\theta\right)\right] \tag{132}$$

Now find $\delta = \frac{\partial U}{\partial F_D}$ = deflection due to applied force P again by the theorem of Castigliano.

$$\frac{\partial U}{\partial P} = \frac{4}{2EI}\Big[0 - M_o r\left[r\left(\frac{\pi}{2}-1\right) - \frac{d}{2}\left(\frac{\pi}{2}+\frac{2}{3}\right)\right] + \frac{P}{2}\left[\left(\frac{3\pi}{4}-2\right)r^3 - r^2 d\left(\frac{5\pi}{8}-\frac{1}{3}\right) + \frac{d^2 r}{4}\left(\frac{5\pi}{8}+\frac{4}{3}\right)\right] \tag{133}$$

$$\delta = \frac{2}{EI}\Big\{\frac{Pr}{2}\left[-r\left(\frac{\pi}{2}-1\right)\left(+\frac{d}{2}\left(\frac{\pi}{2}+\frac{2}{3}\right)\right]\cdot\left[r\left(1-\frac{2}{\pi}\right) - d\left(\frac{1}{2}+\frac{2}{3}\right)\right] + \frac{P}{2}\left[\left(\frac{3\pi}{4}-2\right)r^3 - r^2 d\left(\frac{5\pi}{8}-\frac{1}{3}\right) + \frac{d^2 r}{4}\left(\frac{5\pi}{8}+\frac{4}{3}\right)\right]\Big\} \tag{134}$$

$$\delta = \frac{Pr}{EI}\Big\{-r^2\left(\frac{\pi}{2}-1\right)\left(1-\frac{2}{\pi}\right) + rd\left(\frac{\pi}{2}-1\right)\left(\frac{1}{2}+\frac{2}{3\pi}\right) + \frac{dr}{2}\left(\frac{\pi}{2}+\frac{2}{3}\right)\left(1-\frac{2}{\pi}\right) - \frac{d^2}{2}\left(\frac{\pi}{2}+\frac{2}{3}\right)\left(\frac{1}{2}+\frac{2\pi}{3}\right) + \text{2nd terms}\Big\} \tag{135}$$

$$\delta = \frac{Pr}{EI}\Big\{-r^2\left(\frac{\pi}{2} - 2 + \frac{2}{\pi}\right) + rd\left(\frac{\pi}{4} - \frac{1}{6} - \frac{2}{3\pi}\right) \pm rd\left(\frac{\pi}{4} - \frac{1}{6} - \frac{2}{3}\right) - \frac{d^2}{2}\left(\frac{\pi}{4} + \frac{2}{3} + \frac{4}{9\pi}\right) + r^2\left(\frac{3\pi}{4} - 2\right) - rd\left(\frac{5\pi}{8} + \frac{4}{3}\right) + \frac{d^2}{4}\left(\frac{5\pi}{8} + \frac{4}{3}\right)\Big\} \tag{136}$$

$$\delta = \frac{Pr}{EI}\left[r^2\left(\frac{\pi}{4} - \frac{2}{\pi}\right) - rd\left(\frac{\pi}{8} + \frac{4}{3\pi}\right) + d^2\left(\frac{\pi}{32} - \frac{2}{9}\right)\right] \tag{137}$$

$$\delta = \frac{Pr}{EI}\left[r^2(.149) - rd\,(.818) + d^2(.027)\right] \tag{138}$$

The term in the bracket is not a perfect square. If it is noticed that the last term in the bracket is second order of magnitude small,

some simplification can be obtained by dropping it.

$$\delta = \frac{Pr}{EI}[r^2 (.149) - rd (.818)] \tag{139}$$

$$= \frac{Pr^3}{EI}[.149 - \frac{d}{r}(.818)] \tag{140}$$

Define D_b as the outer diameter of the wave guide which is also the inner ball bearing race. D_b is smaller than the pitch diameter D by the amount of the clearance space of the ball bearings.

$$D_b = k_b D \tag{141}$$

Define D_w as the inner diameter of the wave guide. It is a fraction of the outer diameter.

$$D_w = k_w D_b = k_w k_b D \tag{142}$$

Then if it is assumed that the wave guide is of a simple solid ring construction,

$$I = \frac{\ell t^3}{12} = \frac{\ell (D_b - D_w)^3}{12} = \frac{\ell k_b^3 D^3 (1 - k_w)^3}{12} \tag{143}$$

Also r which has been defined as the mean radius can be written as

$$r \approx \frac{D_w + D_b}{4} = \frac{Dk_b (1 + k_w)}{4} \tag{144}$$

Recalling that $d = \frac{D}{R_D}$, we can rewrite the deflection equation for the wave guide as

$$\delta = \frac{P}{E} \left(\frac{Dk_b[1-k_w]}{4}\right)^3 \frac{12}{\ell k_b^3 D^3 (1-k_w)^3} [.149 - \frac{2}{R_D}(.818)] \tag{145}$$

$$= \frac{P}{E}\frac{3}{16\ell}[.149 - \frac{1.636}{R_D}] \tag{146}$$

The loading on the wave guide is the sum of the flexspline deflection force and the separating force of the gear teeth. $P = F_D + F_R$.

$$F_D = \frac{.59\ dE\ell t^3}{r^3(1 - \frac{3}{R_D} + \frac{3}{R_D^3})} = \frac{4.72\ dE\ell t^3}{D^3(1 - \frac{3}{R_D} + \frac{3}{R_D^2})} \tag{102}$$

Substituting in the expressions for $d = \frac{D}{R_D}$ and $t = D(1-k)$

$$P = \frac{4.72DE\ell D^3(1-k)^3}{D^3(R_D - 3 + \frac{3}{R_D})} = 4.72\ DE\ell \frac{(1-k)^3}{(R_D - 3 + \frac{3}{R_D})} \tag{147}$$

$F_R = \frac{T}{D}\tan\phi$ where ϕ is the tooth angle (30°)

$$= .577\,\frac{T}{D} \tag{148}$$

$$P = 4.72\ DE\ell \frac{(1-k)^3}{(R_D - 3 + \frac{3}{R_D})} + .577\,\frac{T}{D} \tag{149}$$

It can be assumed that the dimensional criteria for the wave guide is that it must have some specified degree of stiffness under the above loading. That is, the deflection must be limited to some specified percentage of the pitch diameter D.

$$\delta = CD$$

$$CD = [4.72\ DE\ell \frac{(1-k)^3}{(R_D - 3 + \frac{3}{R_D})} + .577\,\frac{T}{D}]\frac{3}{16E\ell}[.149 - \frac{1.636}{R_D}] \tag{151}$$

$$\frac{CD^2\ell 16E}{3[.149 - \frac{1.636}{R_D}]} - \frac{4.72\ D^2E\ell(1-k)^3}{(R_D - 3 + \frac{3}{R_D})} = .577\ T \tag{152}$$

$$T = D^2\ell\ \{\frac{C\ 16\ E}{(3)(.577)(.149 - \frac{1.636}{R_D}} - \frac{4.72\ E\ (1-k)^3}{.577(R_D - 3 + \frac{3}{R_D})}\} \tag{153}$$

This is of the general form

$$T = (\text{Constant})D^3(\frac{\ell}{D}) \tag{154}$$

where the constant can be seen to reflect the characteristics of the

materials used, the reduction ratio, the radial stiffness requirements of the wave guide which is dependent on tooth depth, and the thickness of the flexspline. This last factor is dependent on the fatigue limits that are required which in turn is dependent on such things as speed of drive, shock loading, and thermal operating conditions.

Finally the equations for the circular spline can be worked out using the formula given on page 353 of reference [11]. This is an energy method equation and results in a relationship between T, b, and ℓ.

$$\delta = \frac{.149\ Pr^3}{EI} \qquad \text{where} \qquad P = \frac{T}{D}\tan 30° = \frac{.577T}{D}, \quad r = \frac{D}{2} \tag{155}$$

If the same dimensional criteria is assumed here as for the wave guide that deflection is limited to some specified percentage of the pitch diameter then $\delta = CD$.

$$CD = \frac{(.149)(.577T)D^3}{ED\ 8\ell\ \dfrac{D^3(1-k_g)^3}{12}} \tag{156}$$

where k_g is the ratio of the outer to inner diameters of the circular spline

$$D^2\ell\ \frac{2CE(1-k_g)^3}{(3)(.149)(.577)} = T \tag{157}$$

This equation can be written in the form

$$T = (\text{Constant})D^2\ell = (\text{Constant})D^3\left(\frac{\ell}{D}\right) \tag{158}$$

Now it can be seen that the torque capacity of each of the three components of the harmonic gear can be written in the form of

$$T = (\text{Constant})D^3\left(\frac{\ell}{D}\right), \tag{159}$$

Therefore the torque capacity of the entire system can be written in the same form. This agrees with the torque capacity equation actually used by the United Shoe Machinery Corporation. The exact value of the

"constant" is proprietary information, but it is based on the factors considered here plus considerations pertinent to the design of the ball bearings and some empirical factors.

Inspection of this general equation suggests that ℓ may be varied at will. This is not entirely true since there are several considerations which will restrict the range of values that ℓ may assume. To illustrate this point consider the possible restrictions imposed by tooth loading.

If the maximum tooth face loading is defined as σ_T,

$$\sigma_T = C_T \frac{\text{Force}}{\text{Contact Area}} \tag{160}$$

$$\text{Force} = \frac{T}{D} \tag{161}$$

because there are two areas in contact.

$$\begin{aligned} \text{Contact area} &= \text{arc of contact x height} \\ &= .05\pi\, D\ell \end{aligned} \tag{162}$$

5% contact arc is a conservative figure and actual values can be as high as 10%

$$\sigma_T = C_T \frac{T}{D} \frac{1}{.05\pi D\ell} = C_T \frac{T}{.05\pi\, D^2 \ell} \tag{163}$$

$$\ell_{max} = \frac{C_T T}{\sigma_T .05\pi D^2} \tag{164}$$

or

$$\left(\frac{\ell}{D}\right)_{max} = \frac{C_T T}{\sigma_T .05\pi D^3} \tag{165}$$

Some of the other considerations which limit ℓ/D values are ball bearing loading, canting of the face under deflection in the standard cup configuration, torsional wind up across the face of the flexspline, and the onset of torsional buckling in the flexspline cup. In

particular the last two limit ℓ/D to 1 in the bell and inverted bell flexspline configuration. Canting of the face under deflection limits the standard cup flexspline configuration to values of $\ell/D = .2$.

Now write the weight equations for the components. If it is assumed that the components are of solid ring construction the general weight equation for each of them will be of the form

$$W = \gamma_s \pi \, (r_o^2 - r_i^2)\ell \;=\; \frac{\gamma_s \pi D^2}{4} (1 - k^2)\ell \tag{166}$$

where the k is the appropriate thickness ratio. Therefore the total weight of the harmonic gear can be expected to be of the form

$$W = \text{constant } D^2\ell = (\text{constant})D^3 \left(\frac{\ell}{D}\right) \tag{167}$$

However we have shown that the torque capacity of the gear is proportional to $D^3 \left(\frac{\ell}{D}\right)$. It then follows that weight is determined only by torque.

$$W = (\text{Constant})T \tag{168}$$

Note that this presumes that ℓ/D may be varied as necessary, which infers that this equation is valid only in the range of the permissable values of $\left(\frac{\ell}{D}\right)$. It is most difficult to compute the value of the constant without going into a detailed design analysis backed up by the empirical results of a test program. The wave guide and circular spline are normally constructed in the form of circular I beams for maximum stiffness per weight. The weight of the ball bearings must also be considered as well as the effect of bolt holes and other miscellaneous items. For these reasons the weight constant is best determined by empirical measurements.

3.3.2.2 Design Calculations

The results of these calculations agree with those used by the United Shoe Machinery Corporation, patent holders and manufacturers of the harmonic gear. However, there are many other considerations which must be taken into account in the design of these gears. This company has done extensive experimental as well as anlaytical investigations into the design, and they are continuing their research into the development of new types and applications. The stresses in the flexspline have been accurately determined and the design is such that these stresses are kept below the endurance limits of the materials used. Fatigue life of the gears is then not dependent on the flexspline, but rather in the life of the ball bearing races. Since the dimensions are calculated with a safety factor on the endurance limit of the materials, the maximum load that a gear can sustain is several times the rated load. This has favorable implications regarding the shock resistance of the device. The normal mode of failure of the device under heavy overload is torsional buckling of the cup section of the flexspline. This does not release the tooth engagement. That is, for our application, failure of the gear would not release the rudder to let it swing freely. It is possible to design sufficient life into the ball bearings to ensure a twenty-year lifetime in service. However, should spalling of the ball bearing races occur, the gear can be safely continued in operation (although at a high noise level) until it is convenient to replace the bearing.

There are three possible configurations of the flexspline. They are the cup (standard), the bell, and the inverted bell.

The standard cup is limited to an ℓ/D of 1/5 by the canting deflections of the edge, but it is easier to manufacture. Both of the

bell arrangements allow larger ℓ/D values because the shape of the bell causes parallel deflection of the tooth faces. The limit is an $\ell/D = 1$ which is based on the torsional windup across the face width of the gear and by the approach of the torque loading to the critical value for torsional buckling of the bell. The bell arrangements result in a much higher torque capacity and for this reason they will be the ones considered for the steering engine application.

The minimum diameter can be computed using the following formula supplied by United Shoe Machinery Corporation.

$$T = k_m R_D D^3 \left(\frac{L}{D}\right) \tag{169}$$

k_m is a load factor which accounts for strength of the materials (σ_y = 140,000 psi, E.L. 80,000 psi); the factors of safety used (35% σ_y, 1.72 on E.L.). The notch sensitivity factor which is a function of the number of teeth; the modulus of elasticity; and empirical factors. In this case $k_m = 1.715$.

The reduction ratio is taken as 200. Reduction ratios as high as 350 have been achieved for light torque design. The upper limit is determined by the fact that as the ratio gets higher, the teeth become smaller until a point is reached where the radial deflections in the circular spline and wave guide allow the teeth to ratchet. That is the teeth slide over each other under high torque loading. It was felt that a reduction ratio of 200 could be guaranteed as satisfactory. However, if it were possible to build a model and test it, this value could probably be extended to ratios above 250.

The minimum diameter can now be calculated.

$$D = \left[\frac{5 \times 10^6}{(1.715)(200)(1)}\right]^{1/3} = 24.5'' \quad \text{where } \ell/D \text{ is taken as } 1 \tag{170}$$

Weights can be calculated from the following empirical formulas supplied by United Shoe Machinery Corporation.

$$\text{Circular Spline} = .05\ D^3 \text{ per } \ell/D = 1/5 \tag{171}$$

$$\text{Wave Generator} = .03\ D^3 \quad \text{" " " "} \tag{172}$$

$$\text{Flexspline} = .02\ D^3 \text{ (does not vary much with } (\ell/D)) \tag{173}$$

$$\text{Total weight for } \ell/D = 1 \approx .42D^3 = (.42)(1.46\text{x}10^4) = 6{,}140 \text{ lbs} \tag{174}$$

Calculation of lifetime (running hours at maximum load) may now be done.

$$\text{Max Ball Load} = \frac{T}{D}\, 2 \sin(\alpha/2) = P_o \tag{175}$$

where α = arc angle between ball centers. For $2\frac{1}{4}$" balls, outside configuration, there are 31 balls, so $\alpha = 11.6°$.

$$P_o = \left(\frac{5 \text{ x } 10^6}{4}\right)\left(\frac{1}{27.5}\right)\left(2 \sin\left(\frac{11.6}{2}\right)\right) = 8410 \tag{176}$$

Note: Divide $5 \text{ x } 10^6$ max torque by 4 rows instead of the 5 rows to allow for uneven loading distribution on the bearings.

From B^{10} bearing life chart, get 500 hours life at 6300 lbs at 1800 rpm. Then bearing life for this gear is:

$$\text{Life} = 500 \left[\frac{6300}{8410}\right]^3 \frac{1800}{78} = 4{,}880 \text{ hours at max load.} \tag{177}$$

Now calculate the bearing life according to the presently used Bureau of Ships criteria.

At 51% load, the lifetime of this harmonic gear would be:

$$L = 500 \left[\frac{6300}{(.51)(8410)}\right]^3 \frac{1800}{78}$$

$$= (500)(3.18)(23.1) = 36{,}700 \text{ hrs.} \tag{178}$$

Although this is the same order of magnitude as the 50,000 hour criteria,

it is more than 26% less than it. The diameter of the harmonic gear required to meet this standard can be calculated as follows.

$$L = 50{,}000 = 5000\left[\frac{6300}{(.51)P_o}\right]^3 \frac{1800}{78}$$

$$\left[\frac{50{,}000}{(500)(23.1)}\right]^{.333} = \frac{6300}{(.51)P_o} = 1.63 \qquad (179)$$

$$P_o = \frac{6300}{(.51)(1.63)} = 7{,}570 \text{ lbs} \qquad (180)$$

$$D = \frac{T}{4P_o}\ 2\sin(\alpha/\ ) = \frac{(5 \times 10^6)\sin(\frac{11.6}{2})}{(2)(7{,}570)} = 33.4'' \qquad (181)$$

Increasing the diameter of the harmonic gear to this value while keeping $\ell/D = 1$ is easily feasible. The weight will increase, but not as much as equations (167) would indicate because lighter sections can now be used. Although equation (168) indicates that weight would remain constant with T, it is based on the presumption that ℓ/D can vary which is not the case here.

However, there is considerable question as to the exactness of the Bureau of Ships criteria. It is an extremely conservative estimate which is arbitrarily arrived at, and its only justification lies in the fact that it indicates a reasonable order of magnitude that can be used in the absence of actual data. It is entirely probable that the 36,700 hour lifetime will be completely satisfactory for this application. It is obviously a conservative value for a merchant ship and may even prove to be so for military ships if and when actual data becomes available. In any case the lifetime greatly exceeds the normal overhaul interval and replacement of the bearings is not difficult. As has already been pointed out fatigue failure of the bearing races is not catastrophic and the unit can continue in operation for a reasonable

period of time. For these reasons the analysis will proceed under the assumption that this lifetime may prove satisfactory in service rather than unfairly penalize the design on the basis of admittedly arbitrary estimates.

The first possible arrangement of the components of the gear is to bolt the gear on the top of the rudder stock shown in Figure (XIIA). An inverted bell flexspline configuration would be required and the wave guide would be situated internally. Drive would be by means of a bevel gear to the wave guide. This arrangement has two disadvantages, however. The first is that adequate support of the heavy internally situated wave generator would probably prove awkward, and the second is that the circular spline must be provided with a support ring capable of reacting the full five million inch pounds torque.

The second possible arrangement is to locate the components of the gear concentrically around the rudder stock as shown in Figure (XIIB). This would utilize a bell shaped flexspline and an external wave guide. The bell is bolted to the deck opposite the upper bearing in order to minimize possible alignment problems. The platform which supports the drive motor only has to react one two hundreth of the load torque and it can easily support the external wave guide. This arrangement can be expected to be lighter in weight than the first one. For these reasons, the analysis will proceed on the basis of the second arrangement.

It now becomes necessary to evaluate the drive train to the gear. The efficiency is computed according to the following formula provided by United Shoe Machinery Corporation.

FIGURE XXII A

HARMONIC GEAR ARRANGEMENT

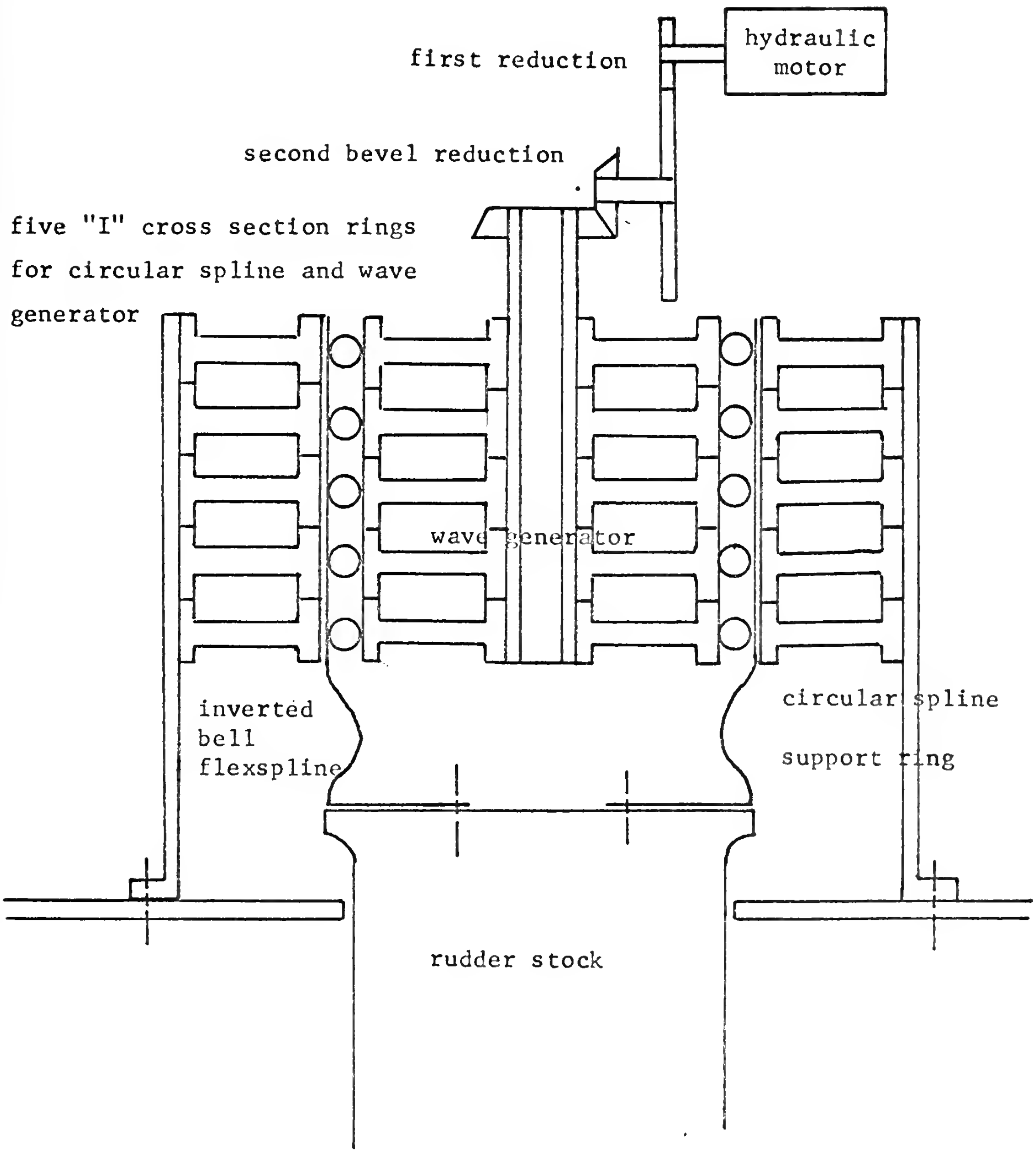
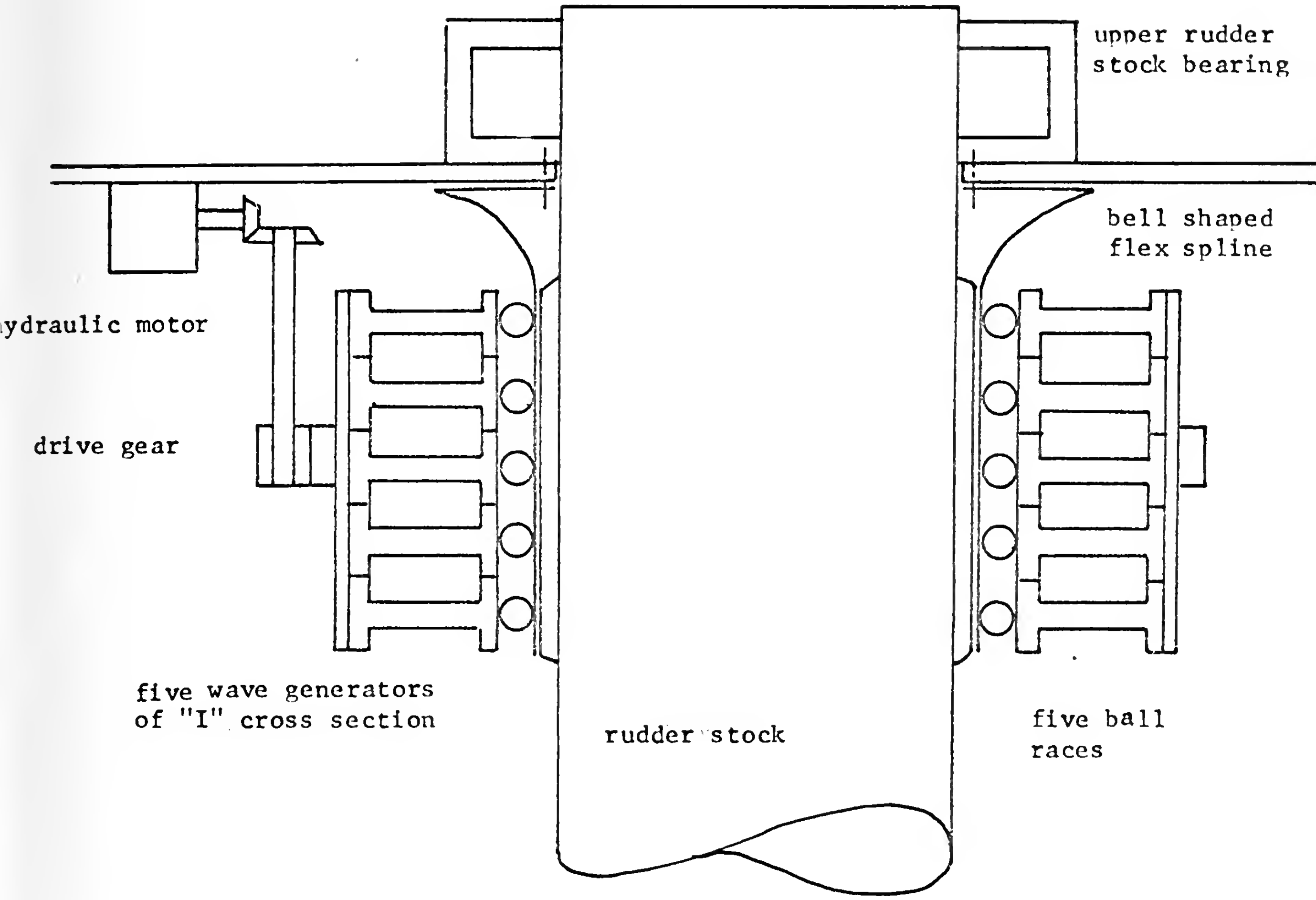


FIGURE XXIIB

HARMONIC GEAR ARRANGEMENT



Efficiency of the gear

$$E = \frac{1}{1 + \frac{R_D}{600}} \tag{182}$$

$$E = \frac{1}{1 + \frac{200}{600}} = \frac{1}{1 + \frac{1}{3}} = \frac{3}{4} = .75\% \tag{183}$$

$$\text{Torque input} = \frac{T_{out}}{E} \frac{1}{R_D}$$

$$= \frac{(5 \times 10^6)}{(.75)(200)} = 3.33 \times 10^4 \text{ in lb @ 78 rpm} \tag{184}$$

Then using a 7.1:1 spur gear second reduction

$$\text{Torque input} = \frac{3.33 \times 10^4}{(.97)(7.5)} = 4{,}580 \text{ in lb @ 585 rpm} \tag{185}$$

Then using a 3:1 bevel gear first reduction

$$\text{Torque input} = \frac{4580}{(.97)(3.0)} = 1{,}575 \text{ in lb or 131 ft. lbs @ 1750 rpm} \tag{186}$$

$$\text{Power} = 1575 \text{ in lb}\left(\frac{\text{ft}}{12 \text{ in}}\right)\left(\frac{1750 \text{ rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{\text{rev}}\right) 3.03 \times 10^{-5} \text{HP/ft lb/min}$$
$$= 43.7 \text{ HP} \tag{187}$$

3.3.2.3 Drive Unit Selection

Now an analysis of the weights of driving mechanisms will be considered. In the selection of electric motors use is made of the standard Bureau of Ships practice of utilizing the normal 50% overload capacity of standard motors.

Weight Comparison of Power Drives. Information is from the catalogues of the indicated manufacturer.

1 Magnetic Coupling to Clutch Control

In this arrangement a 40 H.P. A.C. motor drives through a magnetic coupling to a differential reversing gear. There are two clutches arranged so that in order to drive in a given direction, one clutch engages the appropriate bevel gear in the differential to the drive shaft. A brake is required to prevent the rudder from swinging free when neither clutch is driving.

40 H.P. A.C. motor and magnetic coupling (G.E. Kinatrol)	860 lbs
2 clutches (Stearns size 708 metallic lining)	80 lbs
D.C. rectifier for clutch power (est.)	20 lbs
Brake	100 lbs
Reversing differential gear	25 lbs
	1085 lbs

2 SCR Rectifier and Shunt Wound D.C. Motor

30 HP SCR rectifier (G.E. Speed Variator)	500 lbs
30 HP Shunt wound type D.C. motor (G.E.)	625 lbs
	1125 lbs

Neither a Ward Leonard system nor an M.G. set rectifier was considered because they are larger and heavier than the silicon controlled rectifier.

3 A.C. Motor Plus Hydraulic Transmission

440 V, A.C. squirrel cage motor 40 H.P. (G.E.)	492 lbs
Hydraulic transmission (Hydreco Model 45 pump, model 48 motor)	220 lbs
Oil reservoir and piping	150 lbs
	862 lbs

4 Wound Rotor A.C. Motor

440 V, three phase, 30 HP motor (G.E.)	920 lbs

Although drive #4 is the most compact proposal, it would tend to draw an excessive amount of current. It is possible to design a control system which can switch the rotor resistors in and out of the circuit in order to limit this current to more easily acceptable limits. Design of such a system is complicated and unfortunately time for investigation of it is lacking. Consequently the analysis will proceed using power drive #3. In addition to being the lightest, it is probably the most flexible of all of the drives, and its use will also result in a convenient and compact arrangement. Furthermore, using the hydraulic transmission will in no way detract from the general applicability of the analysis.

The sizes of the first and second reduction gears can be calculated by using the method outlined in reference [23]. For the first reduction of 7.5:1, diameters are fixed and so face width is calculated as follows if spur gear teeth are used.

$D_G = 44.5$ in. $m_G = 7.5:1$ where D_G = Diameter of gear

$d_p = 5.94''$ d_p = Diameter of pinion

$T_p = 4{,}580$ in. lb @ 585 rpm m_G = ratio (188)

$$K = \frac{2T_p}{Fd_p^2}\,\frac{m_G + 1}{m_G}$$ T_p = pinion torque (189)

F = face width

K = rating factor

If use a permissible "K" factor of 500 for this case and take $F/d = 1$

$$F = \frac{2T_p}{Kd^2}\,\frac{m_G + 1}{m_G} = \frac{(2)(4{,}580)}{500\ 5.94^2}\,\frac{8.5}{7.5} = .59'' \tag{190}$$

This "K" is reasonable in view of the intermittent nature of the torque loading. For the second reduction of 3:1 using spur gearing, the diameter and face width of the pinion are calculated as follows:

$m_G = 3:1$, let $K = 500$, $T_p = 1,575$ in lbs @ 1750 rpm let $F/d = 1$,

$$d_p^3 \quad \frac{2T_p}{K} \quad \frac{m_G + 1}{m_G} \tag{191}$$

$$d_p = \frac{(2)(.575)}{500} \quad \frac{5}{4} = 1.98'' \tag{192}$$

$$D_G = 5.96'' \quad F = 1.98'' \tag{193}$$

Or could choose a spiral bevel set from table 13-8 of reference [23].

$d_p = 2.00''$, $D_G = 6''$, $F = 1 \frac{5''}{32}$, width of gear body approx 2"

The weights of the gears can be calculated as follows.

Second reduction (7.5:1)

$$\text{Gear} = \delta_s \pi \frac{(D_o^2 - D_1^2)}{4} \quad F = \frac{(.238)(\pi)}{4}(44.5^2 - 43.5^2)(.59) \quad = \quad 12 \text{ lbs}$$

$$\text{Pinion} = \frac{(.238)(\pi)}{4}(5.94^2)(.59) \quad = \quad 4 \text{ lbs}$$

First reduction (3:1) (using Spiral bevel set)

$$\text{Gear} = (.283)(\pi)\frac{(6^2)}{4}(2'') \quad = \quad 16 \text{ lbs}$$

$$\text{Pinion} = (.283)(\pi)\frac{(2^2)}{4}(2'') \quad = \quad \underline{2 \text{ lbs}}$$

$$34 \text{ lbs}$$

3.3.2.4 <u>Weight Summary</u>

Harmonic gear	6,140 lbs
Drive gear train	34 lbs
A.C. motor plus hydraulic transmission	862 lbs
Drive motor support platform (est.)	100 lbs
Total	7,136 lbs

3.2.2.5 Mathematical Model

A lumped parameter mathematical model may be constructed as shown in Figure XXIII. The parameters of the model must now be calculated.

The harmonic gear has no viscous damping, but it does have a high starting torque loss which remains about constant with time for an initial period. This is due to the requirement of a certain amount of force to deflect the flexspline, even under no load, and the ball bearing "stiction" and frictional losses at low speed. Estimate of starting torque by United Shoe Machinery Corporation is 350 in lbs. A high load, loss torque is the load torque divided by efficiency.

$$
\begin{aligned}
\text{Load torque plus loss torque} &= 3.33 \times 10^4 \\
\text{Load torque} &= (-)\ \underline{2.5 \times 10^4} \\
\text{Loss torque} &= 8{,}300 \text{ in lbs at } \theta_R = 35^\circ
\end{aligned}
\tag{194}
$$

Torque losses can be represented reasonably well by a straight line plot versus load shown in Figure (XXIII). Notice that in this speed range, the losses are proportional to load and are independent of speed.

It is possible to add to this curve the estimated losses in the other gears to better represent the system. This can be represented by the equation

$$T = C + k_L \theta_R \tag{195}$$

$$C = 350 \text{ in lbs}$$

$$k_L = \frac{7950}{35^\circ}\ \frac{180^\circ}{\pi \text{rad}} = 13{,}000 \frac{\text{in lbs}}{\text{rad}}\ (35^\circ = .611 \text{ rad})$$

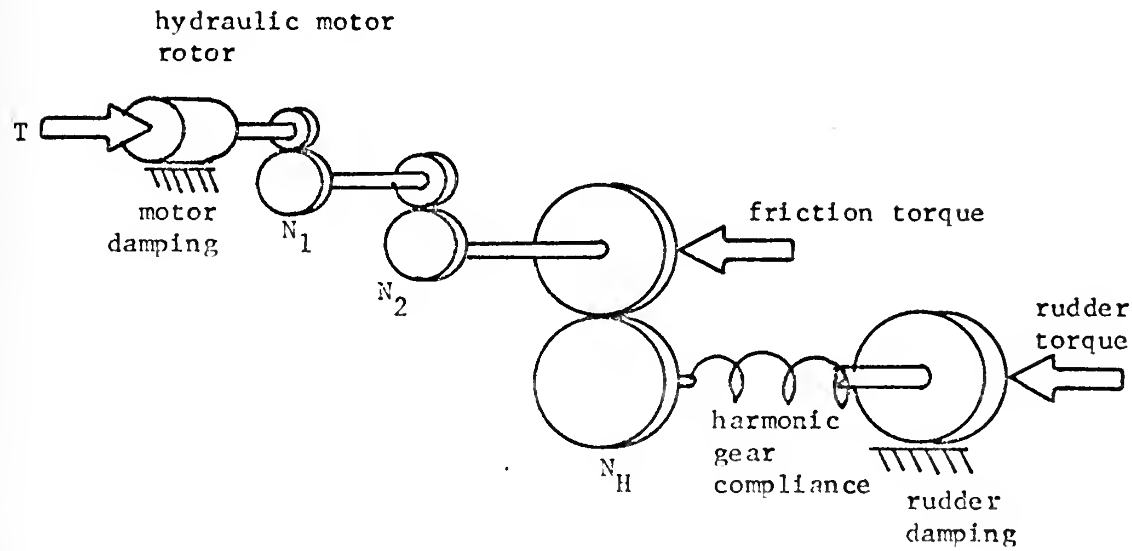
If the load torque is assumed to be representable as a ramp function, then

$$T_L = k_R \theta_R \tag{196}$$

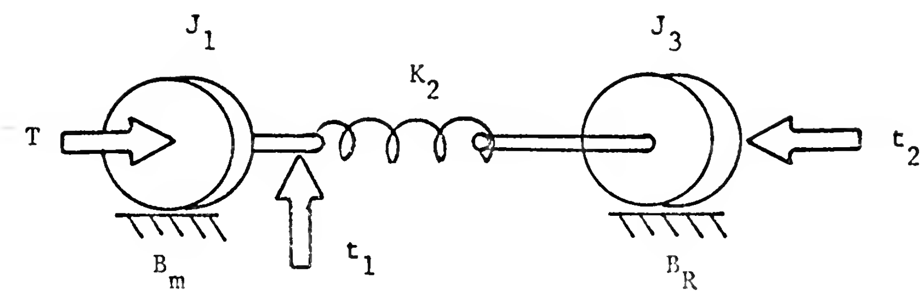
FIGURE XXIII

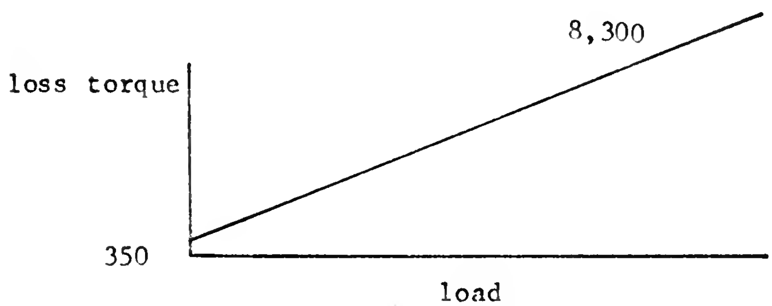
MATHEMATICAL MODEL OF HARMONIC GEAR

System Schematic Diagram



Lumped Parameter Model





torque losses

$$K_R = \frac{5 \times 10^6 \text{in lbs}}{35°} \times \frac{57.3°}{\text{radian}} = 8.19 \times 10^6 \frac{\text{in lbs}}{\text{rad}} \tag{197}$$

$$T_L = (8.19 \times 10^6)\theta_R \tag{198}$$

The stiffness of the harmonic gear can be estimated by use of the following formula provided by United Shoe Machinery Corporation.

$$K_H = 16{,}000 \times D^3 \frac{\text{in lbs}}{\text{radius}} = \text{output stiffness} \tag{199}$$

$$= 16{,}000 \times (24.5)^3 = 245 \times 10^6 \frac{\text{in lbs}}{\text{radian}} \tag{200}$$

The deflection of the gear under load can be calculated now.

$$\theta_\delta = \frac{5 \times 10^6 \text{ in lbs}}{246 \times 10^6 \frac{\text{in lbs}}{\text{radian}}} = 2.03 \times 10^{-2} = .0203 \text{ rad} = 1.165° \tag{201}$$

$$\text{Input stiffness} = \frac{\text{Output stiffness}}{N^2} \tag{202}$$

The term $\frac{\rho\pi}{2}$ appears in all of the inertia calculations and is worked out here for convenience.

$$\rho_{steel} = \frac{489.6 \text{ lbs}}{\text{ft}^3} \frac{\text{ft}^3}{1728 \text{ in}^3} \frac{\text{sec}^2}{32.2} \frac{\text{ft}}{12 \text{ in}} = 7.33 \times 10^{-4} \tag{203}$$

$$\frac{\rho\pi}{2} = \frac{(7.33 \times 10^{-4})(3.14)}{2} = 1.15 \times 10^{-3} \frac{\text{lb sec}^2}{\text{in}^4} \tag{204}$$

$$J_1 = J_m(\text{motor inertia}) + J_{P1}(\text{pinion inertia}) + \frac{J_{G1}(\text{gear inertia})}{N_1^2} + \frac{J_{P2}(\text{pinion inertia})}{N_1^2} + \frac{J_{G2}(\text{gear inertia})}{(N_1N_2)^2} + \frac{J_W(\text{wave guide inertia})}{(N_1N_2)^2} \tag{205}$$

J_m from Appendix (IV) $= .225$ in lb sec^2

$$J_{P1}\text{pinion} = \frac{\rho\pi F r^4}{2}$$

$$= 1.15 \times 10^{-3}(1)(2) \qquad = 2.30 \times 10^{-3} \text{ in lb sec}^2$$

$$J_{G1} = \frac{1.15 \times 10^{-3}(1)(81)}{3^2} = 1.035 \times 10^{-2} \text{ in lb sec}^2$$

$$J_{P2} = (1.15 \times 10^{-3} \frac{(.6)(2.97^4)}{3^2} = 5.9 \times 10^{-2} \text{ in lb sec}^2$$

$$J_{G2} = \frac{(1.15 \times 10^{-3})(24.4)(22.1875^4 - 21.6375^4}{(7.5)^2 \ (3)^2}$$

$$= \frac{(1.15 \times 10^{-3})(24.5)(2.2 \times 10^4)}{(56.2)(9)} = 1.225 \text{ in lb sec}^2$$

Using inertia formula supplied by United Shoe Machinery Corporation the inertia of the wave generator

$$J_W = \frac{Wk^2}{GN_1^2 \ N_2^2} = \frac{5(.05 \ D^3)}{386.4} \ \frac{(.8 \ D)^2}{(7.5)^2(3^2)}$$

$$= \frac{(.25)(.64)(24.5)^5}{(386.4)(56.2)(9)} = \underline{7.24 \text{ in lb sec}^2}$$

$$\text{Total} = J_1 = 8.754 \text{ in lb sec}^2$$

$$K_2 = \frac{K_H \text{ and } K_{Rs}}{N_1^2 \ N_2^2 \ N_H^2} = \frac{\frac{K_{Rs} \ K_H}{K_{Rs} + K_H}}{(3^2)(7.5^2)(200^2)}$$

$$= \frac{228 \times 10^6}{(9)(56.2)(40000)} = 11.25 \ \frac{\text{in lbs}}{\text{rad}}$$

$$J_3 = \frac{J_{\text{rudder \& stock}} + J_{\text{added mass}}}{N_1^2 \ N_2^2 \ N_H^2} = \frac{1.12 \times 10^5 + 6.57 \times 10^5}{(N_1 N_2 N_H)^2}$$

$$= \frac{7.69 \times 10^5}{20.25 \text{x} 10^6} = .038 \text{ in lb sec}^2$$

$$B_R = \frac{\text{Rudder Damping}}{(N_1 N_2 N_H)^2} = \frac{22.1 \times 10^6}{20.25 \text{x} 10^6} = 1.092 \text{ in lb sec}$$

B_m from Appendix (VI) $= .216$ in lb sec

$$t_1 = \frac{C + K_L \theta_R}{N_1 N_2} = \frac{350 + 13000\theta_R}{22.5} = 15.6 + 578\theta_R = 15.6 + \frac{578\theta_R}{4500\,\theta_R/\theta_1} = 15.6 + .1286\theta_1$$

$$t_2 = \frac{K_R\ \theta_R}{N_1N_2N_H} = \frac{8.19\text{x}10^6\theta_R}{4500} = (1.82\text{x}10^3)\theta_R = \frac{(1.82\text{x}10^3)\theta_R}{4500\theta_R/\theta_3} = .404\theta_3$$

Examining the right half of this mathematical equivalent system we see that a comparatively weak spring connects a heavily damped light inertia and a large load to the drive units. The combination of light inertia and heavy damping preclude any significant transient effects in the output. Hence it can be seen that the spring will react primarily to the large load torque. That is

$$K_2(\theta_1 - \theta_3) = t_2 \tag{206}$$

Since t_2 is a linear ramp function of θ_3 then the equation becomes

$$K_2(\theta_1 - \theta_3) = .404\theta_3 \tag{207}$$

and the system can be simplified to the model shown in Figure (XXIV).

Notice that the weak spring K_2 in effect isolates the heavy damping from the mechanical drive section of the system.

The system equation is

$$T = J_1\theta_1'' + B_m\theta_1' + K_2(\theta_1 - \theta_3) + t_1 \tag{208}$$

$$T = J_1\theta_1'' + B_m\theta_1' + t_2 + t_1 \tag{209}$$

If the step nonlinearity in the friction torque t_1 is neglected, it can be written as

$$T = J_1\theta_1'' + B_m\theta_1' + .404\theta_3 + .129\theta_3 \tag{210}$$

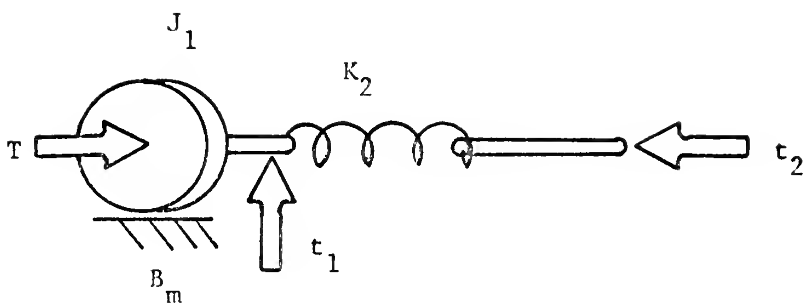
Solve the spring equation for θ_1

$$K_2\theta_1 - K_2\theta_3 = .404\theta_3 \tag{207}$$

$$\theta_1 = \frac{(.404 + K_2)}{K_2}\theta_3 = \frac{11.654}{11.25}\theta_3 \tag{211}$$

FIGURE XXIV

SIMPLIFIED MODEL



Lumped Parameter Model Reduced by Simplifying Assumptions

Write the T equation in terms of θ_3

$$T = (8.754)(\frac{11.654}{11.25})\theta_3'' + \frac{(.216)(11.654)}{11.25}\theta_3' + .533\ \theta_3$$

$$= 9.06\ \theta_3'' + .224\ \theta_3' + .533\ \theta_3 \qquad (212)$$

This can be written in terms of θ_R by using $\theta_R = \frac{\theta_3}{4500}$

$$T = 40800\ \theta_R'' + 1009\ \theta_R' + 2400\ \theta_R \qquad (213)$$

$$= A\theta_R'' + B\theta_R' + C\theta_R \qquad (214)$$

The next step is to combine these equations with the hydraulic motor equations which are derived in Appendix (VI).

$$D_m\dot{\theta}_m = C_p\ x - \frac{C_L}{D_m}T \qquad (215)$$

$$T = \frac{C_p D_m}{C_L}x - \frac{D_m^2}{C_L}\dot{\theta}_m \qquad (216)$$

$$T = \frac{C_p D_m}{C_L}x - \frac{D_m^2}{C_L}(\frac{K_L + K_2}{K_2}N)\dot{\theta}_R \qquad (217)$$

The values of the constants in the hydraulic equations are calculated in Appendix (VI). Substituting those values into the above equation results in the following:

$$T = \frac{(515)(.772)}{.005}x - \frac{(.772)^2}{.005}\frac{11.654}{11.25}4500\ \dot{\theta}_R$$

$$= 79500\ x - 5.55 \times 10^5\ \dot{\theta}_R$$

$$= 79500\ x - 550{,}000\ \dot{\theta}_R \qquad (218)$$

Now equate to equation (213) for T

$$79500\ x - 5.55 \times 10^5\theta_R = 40800\ \theta_R'' + 1009\theta_R' + 2400\theta_R$$

$$79500\ x = 40800\ \theta_R'' + 556009\ \theta_R' + 2400\ \theta_R \qquad (219)$$

$$x = .514\,\theta_R'' + 7.00\ \theta_R' + .302\ \theta_R \qquad (220)$$

Now try position and rate feedback

$$x = C_1\varepsilon + C_2\varepsilon' \text{ where } \varepsilon = \theta_R^* - \theta_R = \text{error signal} \tag{221}$$

Using operator notation $D = \frac{d}{dt}$

$$(C_1 + C_2D) = (.514\ D^2 + 7.00\ D + .302)(\theta_R^* - \varepsilon) \tag{222}$$

$$\frac{\varepsilon}{\theta_R^*} = \frac{(.514\ D^2 + 7.00\ D + .302)}{(.514\ D^2 + (7.00 + C_2)D + (.302 + C_1)} \tag{223}$$

Steady state error is 1/4° in 35°. This determines the value of C_1

$$\frac{\varepsilon}{\theta_R^*} = \frac{.25°}{35.0°} = .00714 = \frac{.302}{.302 + C_1} \tag{224}$$

$$C_1 = \frac{.302(1 - .00714)}{.00714} = \frac{(.302)(.99286)}{.00714} = 42 \tag{225}$$

Now write transfer function of system

$$(42 + C_2D)(\theta_R^* - \theta_R) = (.514\ D^2 + 7.00 + .302)\theta_R \tag{226}$$

$$\frac{\theta_R}{\theta_R^*} = \frac{(42 + C_2D)}{.514\ D^2 + (7.00 + C_2)D + (42 + .302)} \tag{227}$$

Now can solve this for C_2 by imposing the requirement that the damping constant $\xi = .7$[27].

$$\omega_n^2 = \frac{42 + .302}{.514} = 82.4,\ \omega_n = 9.07 \tag{228}$$

$$2\xi\omega_n = (2)(9.07)\xi = 6.94 + C_2 \tag{229}$$

$$\xi = \frac{6.94 + C_2}{18.14} = .7 \tag{230}$$

$$C_2 = (.7)(18.14) - 6.94 = 12.6 - 6.94 = 5.66 \tag{231}$$

Without the rate feedback the damping constant of the system would have been

$$\zeta = \frac{6.94}{(2)(9.07)} = .383 \qquad (232)$$

Although this is lower than normally desired, it is within tolerable limits. It must also be remembered that this analysis neglects the non-linear no load friction torque of 350 in lbs in the harmonic gear. This torque can be expected to reduce the oscillations around the zero point. It can, therefore, be concluded that the harmonic gear steering engine can be controlled satisfactorily with only position feedback when a hydraulic transmission is used.

The above model assumes that the power source (the hydraulic motor) can provide proportional control at any power level required. In actuality the motor saturates quite quickly. In order to investigate the effects of this saturation on stability, a describing function analysis is appropriate. This will determine if a limit cycle is possible under some drive conditions.

Assumptions required for describing function validity are:[28]

(1) System is autonomous (i.e. unforced and time invariant).

(2) The nonlinearity is separable and time invariant.

(3) The linear transfer contains sufficient low pass filtering to warrant excluding from consideration the harmonics in the output.

In order to satisfy the requirements of:

(1) θ_R^* must = 0, i.e. order rudder to amidships position

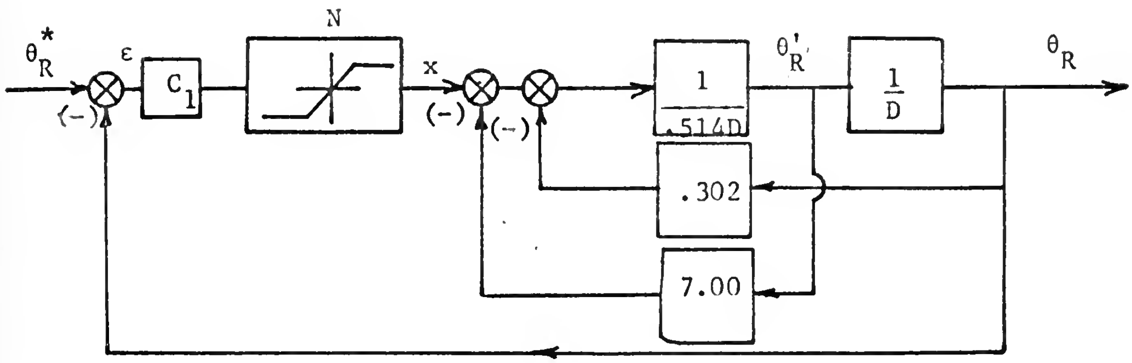
(2) is satisfied

(3) is satisfied because the system is sufficiently damped.

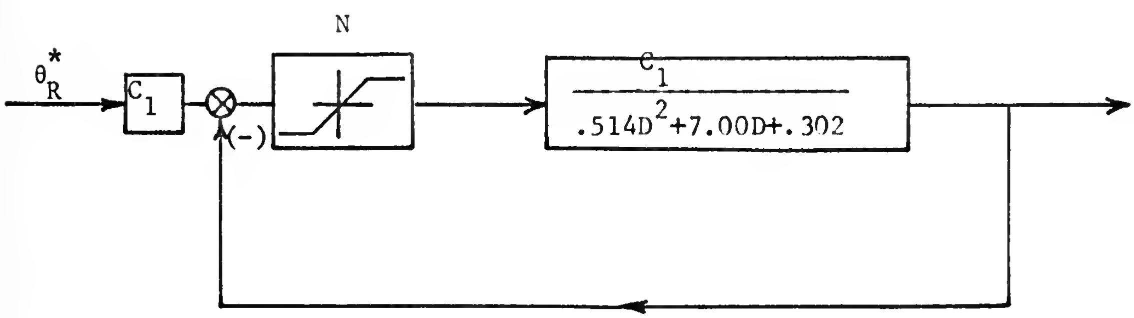
The requirement of (1) which is to examine the system for possible oscillations when $\theta_R^* = 0$ is certainly logically justified because at any other θ_R^* the rudder will have a high load torque which will prevent any

FIGURE XXV

BLOCK DIAGRAM FOR
DESCRIBING FUNCTION ANALYSIS



System Diagram with Saturating Nonlinearity



Describing Function Diagram

oscillation from occurring.

The block diagram of the system to be treated in the describing function analysis is shown in Figure (XXV). The saturating nonlinearity is shown with a slope of $n_1 = 1$ because slope is already accounted for in C_1 and C_2. The analysis proceeds in accordance with Chapter (9) of reference [28].

$$K_{eq} = g + ib = \frac{n_1}{\pi}(2\theta_2 - \sin 2\theta_2) + \frac{4M}{E}\cos\theta_2 + i(0) \tag{233}$$

$$\theta_2 = \sin^{-1}\frac{b}{E},\ n_1 = 1,\ b = .244 \text{ radians},\ M = .244 \text{ radians} \tag{234}$$

$$K_{eq} = \frac{1}{\pi}(2\sin^{-1}\frac{b}{E} - \sin(2\sin^{-1}\frac{b}{E})) + \frac{4M}{E}\cos\sin^{-1}(\frac{b}{E}) \tag{235}$$

$$K_{eq} = \frac{1}{\pi}(2\sin^{-1}\frac{b}{E} - 2(\frac{b}{E})\sqrt{1-(\frac{B}{E})^2}) + \frac{4M}{\pi E}\sqrt{1-(\frac{b}{E})^2} \tag{236}$$

$$= \frac{2}{\pi}[\sin^{-1}\frac{b}{E}\sqrt{1-(\frac{b}{E})^2} + \frac{2M}{E}\sqrt{1-(\frac{b}{E})^2}] \tag{237}$$

$$= \frac{2}{\pi}[\sin^{-1}\frac{b}{E} + (\frac{2M-b}{E})\sqrt{1-(\frac{b}{E})^2}] \tag{238}$$

but $b = M$ because the slope $= 1$, so have

$$K_{eq} = \frac{2}{\pi}[\sin^{-1}(\frac{b}{E}) + \frac{b}{E}\sqrt{1-(\frac{b}{E})^2} \tag{239}$$

where E comes from letting $e_R = E \sin \omega t$.

The condition for a sustained oscillation is

$$G = -\frac{1}{K_{eq}} \tag{240}$$

Set $D = j\omega$ in the system transfer function and investigate locus of $G(j\omega)$

$$G = \frac{42}{.514D^2 + 7.00D + .302} = \frac{42}{(.514)(-\omega^2) + 7.00(j\omega) + .302}$$

$$= \frac{42(.302 - .514\,\omega^2) - j(42)(7.00)\omega}{(.302 - .514\,\omega^2)^2 + 7.00^2\,\omega^2} \tag{242}$$

Look at $-\frac{1}{K_{eq}}$ for various values of E

for E<b, The describing function is not valid in this case for values of E<b. That is, the system is in the proportional control mode of operation and the previous analysis without the describing function is valid.

for E=b,
$$-\frac{1}{\frac{2}{\pi}[\sin^{-1}(1)+1\sqrt{1-1}]} = -\frac{1}{\frac{2}{\pi}[\frac{\pi}{2}+0]} = -1 \qquad (243)$$

for E=∞,
$$\frac{1}{\frac{2}{\pi}[\sin^{-1}(0) + 0 \quad 1-(0)]} = -\infty \qquad (244)$$

Therefore the range of values of $-\frac{1}{K_{eq}} = -1$ to $-\infty$. Now go back to equation for G to see if its amplitude is less than -1 when its trace crosses the real axis.

$$\text{Im part of G} = \frac{(42)(7.00)\omega}{(.302 - .514\,\omega^2)^2 + (7.00^2)\omega^2} = 0 \qquad (245)$$

Inspection of this equation shows that it is satisfied when $\omega = 0$ and when $\omega \to \infty$.

To get some idea of the behavior of the locus, plot some indicator points.

$$\text{Real G} = \frac{42(.302 - .514\,\omega^2)}{(.302 - .514\,\omega^2)^2 + 7.00^2\,\omega^2}$$

$$\text{for } \omega = 0,\ \text{Re G} \quad \frac{42(.302 - 0)}{(.302 - 0)^2 + 0} = \frac{42}{.302} = 139$$

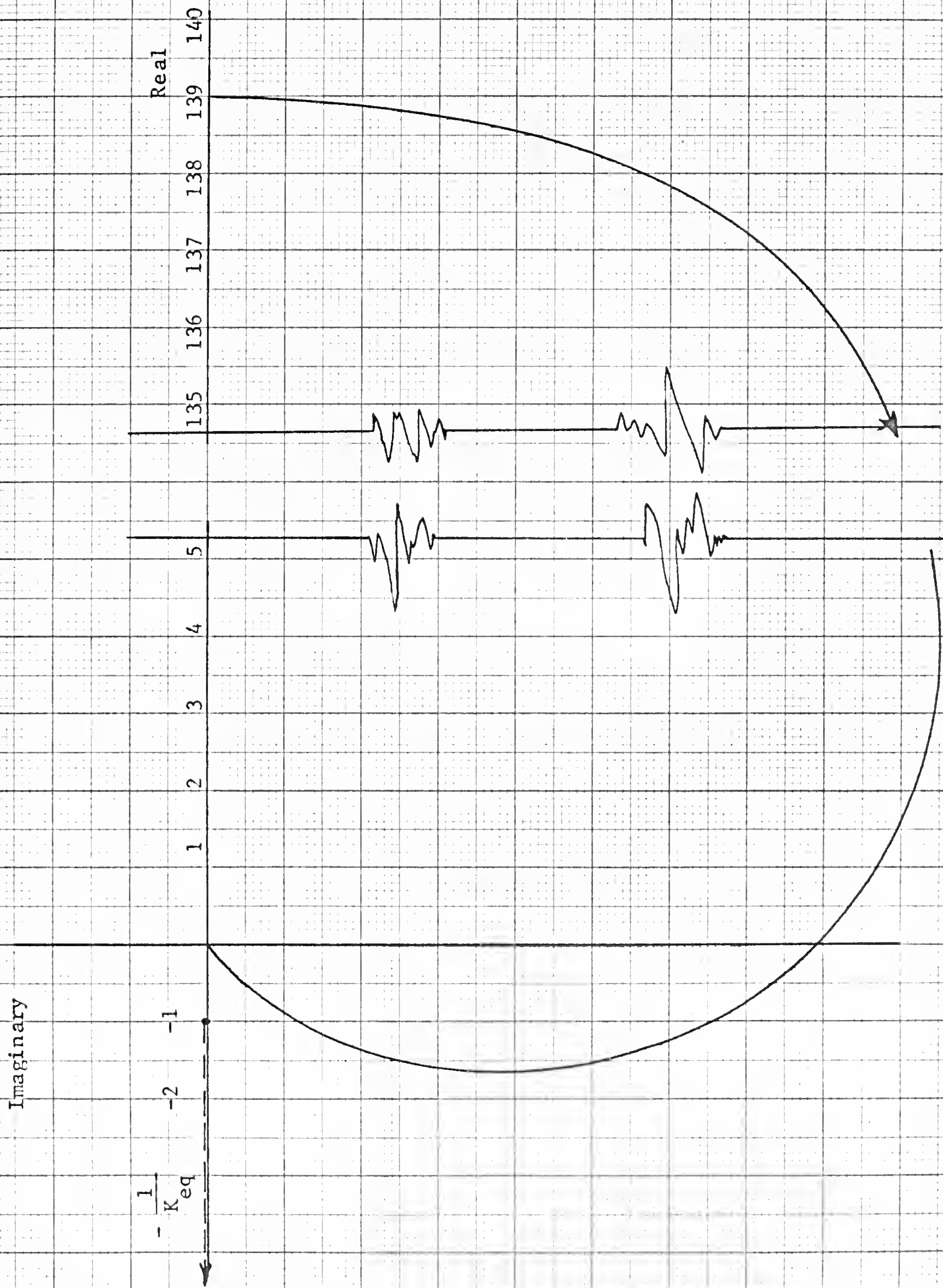
$$\text{for ReG} = 0, \quad 0 = \frac{42(.302 - .514\,\omega^2)}{(\ )^2 + (\ )^2}$$

$$\omega^2 = \frac{.302}{.514} = .589$$

$$\omega = .766$$

FIGURE XXVI

PLOT OF $-\frac{1}{K_{eq}}$ VERSUS G



$$\text{Im } G = \frac{(-)(42)(7.00)(.766)}{[.302 - (.514)(.589)]^2 + [(7.00^2)(.589)]} = -\ 7.82$$

Can see from plot in Figure (XXVI) that no limit cycle is possible with this model. In view of the wide separation of the traces it is probable that the actual system equation which is of higher order than this model will also not intersect the $-\frac{1}{K_{eq}}$ trace. In fact one of the simplifications of this model was to remove the frictional starting torque of 350 in lbs which can be expected to discourage any oscillations of the drive end of the system.

It is concluded that high gain position feedback is required to meet the requirements for static error. Rate feedback is not required if it is possible to accept some amount of oscillation in the drive end. No instabilities are indicated in the control system. Therefore a position feedback control system used with a hydraulic transmission is a feasible method of powering and controlling the harmonic gear steering engine.

3.3.2.6 Compatibility

The harmonic gear physical arrangement fits easily into the space normally allocated for steering engines. In fact its concentric arrangement and moderate height prove to be a neat and compact steering engine "package". It provides an unlimited arc of travel for the rudder, and its rates of travel could be greatly increased. The control required can be conveniently provided in a self contained unit.

However, the large compliance of the harmonic gear does introduce another area that must be considered carefully. The natural frequency of the rudder, rudder stock, and harmonic gear are calculated as follows.

$$\omega_n = \frac{K_{Rs} \text{ and } K_H}{J_{\text{rudder \& stock}} + J_{\text{added mass}}} = \frac{228 \times 10^6}{7.69\text{x}10^5} = 17.25 \ \frac{\text{rad}}{\text{sec}}$$

$= 2.75$ cps or 165 cpm

Although this is well within the range of propeller blade excitation frequencies, a more probable source of trouble will be flutter excitation of a hull vibration mode. McGoldrick[29] has explored the phenomena in detail in his investigation of the DD 931 vibration problem. The high damping calculated in Appendix (I) will not be available to reduce such an oscillation because this damping is frequency dependent. In fact the existence of the flutter phenomenon depends upon the characteristic that fluid damping decreases and may even become negative at certain frequencies. Prediction of this behavior is closely related to the details of the rudder, flow velocities around it, and the relative location of the rudder stock[30]. As a result, each rudder must be treated individually. The other part of the problem is the interaction of the hull with the rudder. In the DD 931 vibration, the flutter condition reduced the overall damping of the combined hull-rudder system to the point where the system became sensitive to wave excitation at the bow.

There are several general points which emerge from the above investigation. Most hulls have several large amplitude natural modes of various types of vibration which occur principally in the frequency range below 600 cpm. Coupling of the rudder torsional vibration mode with one or more of the hull modes is possible and may be expected. Rudder vibration will generally reduce its own damping and may couple with the hull in such a way as to reduce the damping of the overall system. There are sufficient excitation sources available that a vibration will result if the appropriate coupling and reduction of

damping occurs. The natural frequency of the rudder controls the existence and amplitude of its vibration and hence the possible magnitude of its coupling effects.

These considerations have led McGoldrick to suggest that a lower limit of 600 cpm be selected as a natural frequency criterion for rudder design. This is not to infer that should a rudder be installed with a natural frequency below this range a vibration will automatically ensue. Further even if such a vibration does occur, there is a good possibility that it can be corrected. This was done in the case of the DD 931 by changing the toe-in of the rudder which sufficiently detuned the system to stop the vibration. Unfortunately there is insufficient information to guarantee the effectiveness of this cure in every case. At any rate it is clear that a careful investigation of the particular installation is in order when the natural frequency of the rudder system is in this range.

The harmonic gear will probably result in these low frequencies in most rudders in the medium and high torque range. Although the stiffness of the harmonic gear increases rapidly with diameter, examination of equation (252) shows that the combined stiffness of the rudder and stock would have to be increased by a factor of more than 1000 to place it above McGoldrick's criteria. Probably only in the low torque range will the inertias be low enough and the relative diameters of the gear be large enough to warrant neglect of the vibration problem.

3.3.3 Ball Bearing Screw

3.3.3.0 General

The ball bearing screw has many advantages when used as a steering engine, the major one being that it is an efficient high reduction ratio

device. It is a linear actuator and its linear motion must be converted to rotary motion by means of one of the three mechanisms discussed in connection with the hydraulic piston and cylinder. Once again the rapson slide with its low travel requires the smallest actuator. Consequently the analysis will concern itself with an arrangement in which ball bearing actuators work through a rapson slide to provide the rudder torque.

In considering the design parameters pertinent to the selection of a ball bearing screw for this application, fatigue life immediately emerges as the most important consideration. Rudder rates are so low that traverse velocities are well below critical speeds. Because the travel is comparatively short and the ball bearing units required are large, column buckling is not usually a consideration. Only for very large values of nominal radius R does the longer lever arm reduce the required force to the point where the smaller diameter units that can be used approach buckling conditions for these longer travels.

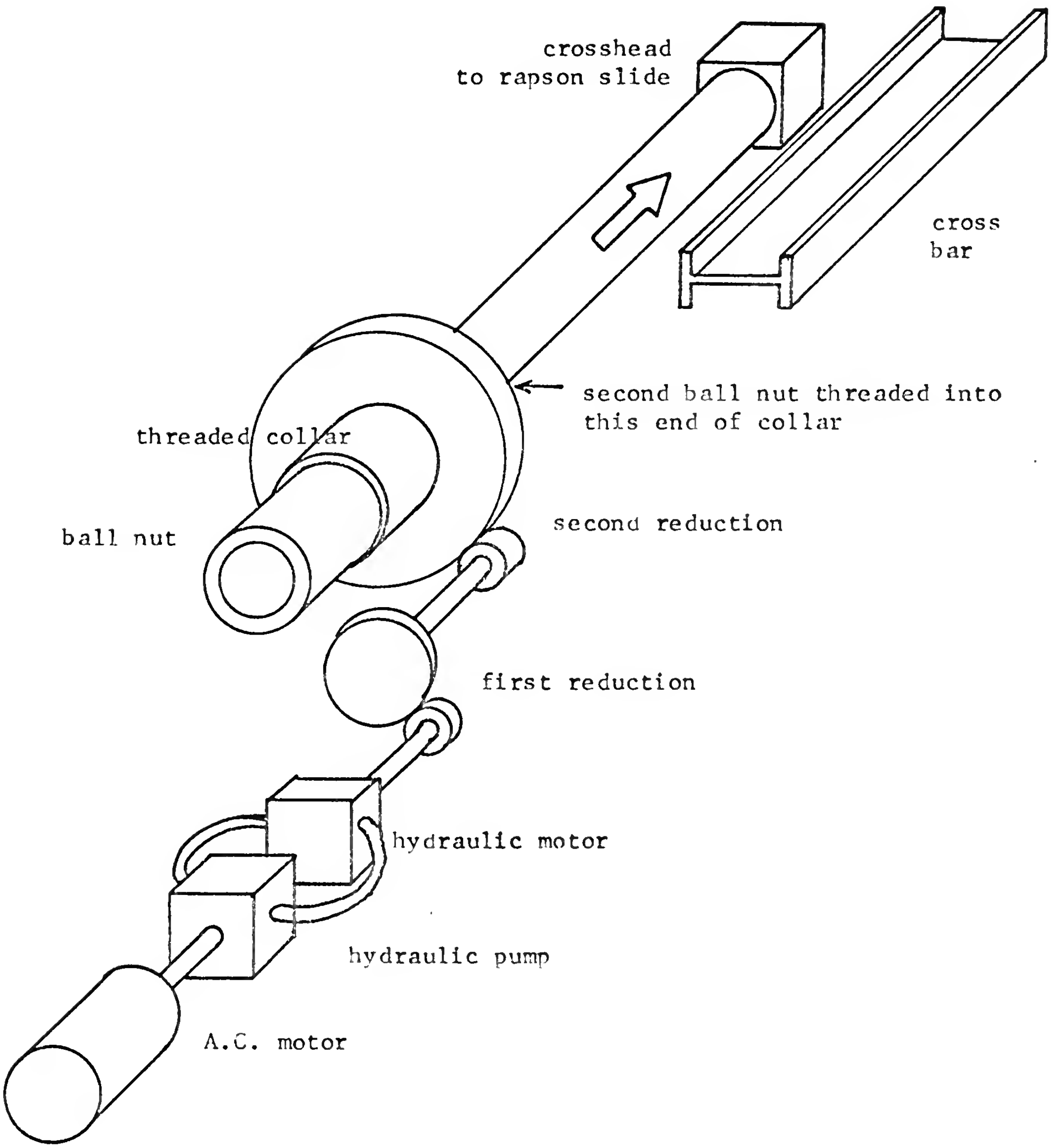
The capacity of the ball bearing is primarily a function of size and hardness of material. The practicable upper limit to ball diameter is about 2.5 inches. Above this value it is extremely difficult to obtain the desired hardness through out the material[26]. However, ball sizes used in the largest available ball bearing screws do not even approach this limit. Rather, the upper limit on size is dictated by manufacturing considerations. The equipment presently available to manufacture ball bearing screws is limited to a ten inch diameter[31]. It is important to recognize that merely increasing the diameter of the balls and the unit is not a cure all because in so doing, new problems are introduced. As ball size is increased, larger diameter shafts and larger leads are required. The first of these serves to increase weight because

far more material is present in the shaft then is necessary to support the axial or buckling loads. Using hollow shafts offers a partial solution, but the weight still rises somewhat and manufacturing problems are introduced. Use of larger leads reduces the effective reduction ratio for the device. In as much as its principal attraction for this application is that it is a high reduction ratio mechanism, increasing lead tends to detract greatly from its usefulness. In the present arrangement, larger leads require higher reduction ratios in the drive train to the screw. Further the drive gearing to the screw operates at much higher torques which require that the gear dimensions and hence weight be larger. One other solution to the problem is to increase the number of rows of circulating balls. This effectively divides the load over a larger contact area, thus increasing lifetime. As the number of rows becomes large, distributing the load evenly among all the balls becomes a problem. Of course the point can also be reached where the lengths of the ball nuts become inconveniently large.

The specific arrangement of the ball bearing screw in a steering engine that will be considered in this analysis is shown in Figure (XXVII). The choice of the rapson slide to convert the linear motion to rotary motion has already been discussed. The same rapson slide used in the electro-hydraulic analysis will be used here in order to facilitate comparisons. There are two possible arrangements of the ball bearing screw. One is to locate the ball nut at the end and rotate it with the drive gearing. The shaft does not turn and is rigidly connected to the slide block. The thrust bearing is incorporated in the bearings for the ball nut. This arrangement requires a crossbar. The second arrangement locates the ball nut at the slide block and the shaft is driven by the

FIGURE XXVII

BALL BEARING SCREW ARRANGEMENT



drive gearing. Thrust and radial bearings can be located at both ends of the shaft. A crossbar is still felt to be necessary. Although there is probably sufficient material in the shaft to support the bending from the reaction force F_R this force would further load the balls. This would require increasing the size of the unit considerably because, as has been shown, the loading of the balls is the critical part of this design. Because both designs require the crossbar, the differences in weight will be small. The first proposal is the lightest because the drive gear is integral to the ball unit and the slide block can be made a little smaller. Although the second design is much more resistant to column buckling because of its bearings at both ends of the shaft, this mode of failure is probably not important. Therefore the lighter first design is chosen.

3.3.3.1 <u>Design Calculation</u>

The largest ball bearing screw for which information was readily available is the Saginaw six inch model with 10.5 turns of balls and a lead of 1 inch[32]. The same rudder stock size as that used in the previous calculations was chosen for comparison purposes. Calculations for the smallest rapson slide nominal radius of nineteen inches are as follows.

$R = 19''$

$F_p = 201{,}000$ lbs (includes friction forces in the slide)

Using the Bureau of Ships lifetime criteria,

$P_{\omega t} = .51\ F_p = 102{,}500$ lbs

Travel $= 26.2$ in

Traverse velocity $= \dfrac{26.6 \text{ in}}{30\text{sec}} = .886$ in/sec

Drive speed for a lead of 1 inch = $\frac{(.886)(60)}{(1)}$ = 53.46 rpm

The operating load for 1×10^6 revolutions life C = 258 405 lbs.

Using two units and assuming uniform load distribution, the expected life can be computed as follows.

$$L = \left(\frac{C}{P_{wt}}\right)^3 = \left(\frac{258405}{(.5D(102,500)}\right)^3 = 128 \times 10^6 \text{ revolutions life}$$

This corresponds to

$$\frac{128 \times 10^6}{(53.6)(60)} = 39,800 \text{ hours.}$$

Although this figure is less than the standard of 50,000 hours, it is within the relm of reasonable values, and it may well prove to be acceptable.

Both ball nuts are mounted on the same shaft. They are threaded into a collar (with set screws) which forms the base of the drive gear. They are pretensioned in the collar, one against the other, so that backlash is eliminated. If the power source is at 1750 rpm, a drive train reduction ratio of 32.7 to 1 is required. This can be achieved by using two spur gear reductions. Arbitrarily assume the first reduction from the motor to be 7:1 and the reduction ratio to the ball nut to be 4.67:1. The torque required to drive the ball nuts is calculated by the formula

$$T = \frac{F_p \text{ Lead}}{2 \text{ efficiency}} = \frac{(201,000)(1)}{(2)(.9)} = 35,600 \text{ in lbs}$$

pinion torque $T_p = \frac{35600}{(4.67)(.98)} = 7,760$ in lbs.

If use a gearing "K" factor of 500 and an F/d = 1.0, the pinion diameter can be calculated.

$$d_p^3 = \frac{2T_p}{K} \frac{m_g + 1}{m_g} = \frac{(2)(7,760)}{500} \frac{5.67}{4.67} = 37.7$$

$$d_p = 3.35 \text{ in}$$

$$F = 3.35 \text{ in}$$

$$D_G = 15.62 \text{ in} \tag{264}$$

Now calculate the gear sizes for the 7:1 reduction ratio. The pinion torque is

$$T_p = \frac{7760}{(7)(.97)} = 1141 \text{ in lbs} \tag{265}$$

Using k = 500 and F/d = .5, d_p can be determined.

$$d_p^3 = \frac{(2)(1141)}{(.5)(500)} \; \frac{8}{7} = 10.42 \tag{266}$$

$$d_p = 2.18 \text{ in}$$

$$F = 1.09 \text{ in} \tag{267}$$

$$D_G = 15.25 \text{ in}$$

Power input may now be calculated

$$p = 1141 \text{ in lbs}\left(\frac{\text{ft}}{12 \text{ in}}\right)\left(\frac{1750 \text{ rev}}{\text{min}}\right)\left(\frac{2\pi \text{radians}}{\text{rev}}\right)\left(3.03\text{x}10^{-5} \; \frac{\text{HP}}{\text{ftlb/min}}\right)$$

$$= 31.7 \text{ H.P.} \tag{268}$$

3.3.3.2 Calculation of Weights

Shaft = $\gamma \pi r^2 \ell$ = $(.283)(\pi)(3^2)(36.18 + 36)$	580 lbs
Ball nuts = $.283\pi(4.25^2-3^2)(36)$	290 lbs
1st reduction pinion $.283\pi \; \frac{3.35^2}{4} \; 3.35$	9 lbs
1st reduction gear $.283\pi \; \frac{15.62^2}{4} \; 3.35$	182 lbs
2nd reduction pinion $.283\pi \; \frac{2.18^2}{4} \; 1.08$	2 lbs
2nd reduction gear $.283\pi \; \frac{15.25^2}{4} \; 1.08$	56 lbs
Gear box and bearings (est.)	250 lbs
Hydraulic pump motor transmission (Hydreco)	220 lbs
Oil reservoir and piping	150 lbs
30 HP 3 phase 440 volt A.C. motor (G.E.)	410 lbs
Total weight of drive	2149 lbs

Rapson slide (same weight as for electro-hydraulic) 1539 lbs

Total weight 3688 lbs

3.3.3.3 Mathematical Model

The system is now sufficiently defined to permit a control analysis. An equivalent system, shown in Figure (XVIIIA), may be calculated. Values for the constants are as follows.

K_1 = equivalent spring constant of the ball bearing shaft. The shaft is loaded in either tension or compression which will cause a linear distortion. This linear distortion of the shaft is reflected in a rotational displacement of the ball nut. The ball bearing screw spring constant is the load torque divided by the angular deflection under this load. Then the ball bearing spring constant must be related back to K_1 in the equivalent system by dividing by the square of the reduction ratio. The calculation is as follows

$$\text{linear distortion in shaft} = e = \frac{F_p}{AE} \tag{269}$$

$$\text{angular displacement in ball nut} = \psi = \frac{e(\text{in})}{\text{lead}(\text{in/rev})}\, 2\pi \left(\frac{\text{radians}}{\text{rev}}\right)$$

$$= \frac{2\pi e}{\lambda} \tag{270}$$

$$K_{BB} = \frac{T_{BB}}{\psi} = \frac{T_{BB}\lambda}{2\pi e} = \frac{T_{BB}\lambda}{2\pi}\;\frac{AE}{F_p \ell} \tag{271}$$

$$K_{BB} = \frac{(35600)(1)\pi(2.587^2)(30\times10^6)}{(2\pi)(201000)(26.6)} = 6.7\times10^5 \text{ in lbs/radian} \tag{272}$$

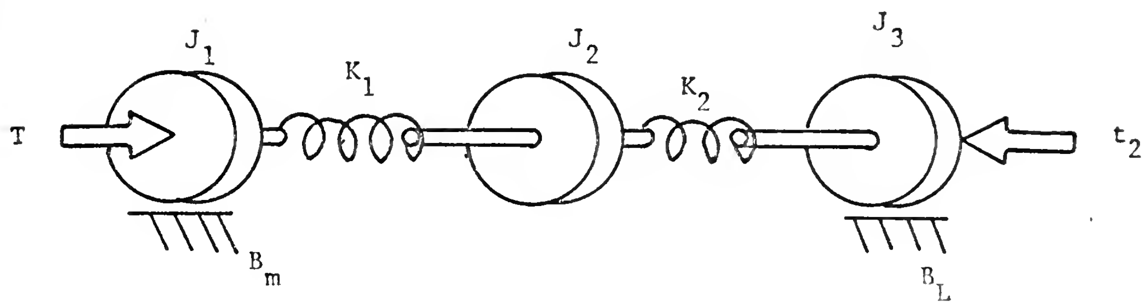
$$K_1 = \frac{K_{BB}}{(N_1N_2)^2} = \frac{6.7\times10^5}{32.7^2} = 627 \text{ in lbs/radian} \tag{273}$$

$$J_3 = \frac{J_{\text{rudder \& stock}} + J_{\text{added mass}}}{(N_1N_2)^2} = .038 \text{ in lb sec}^2 \tag{274}$$

FIGURE XXVIII

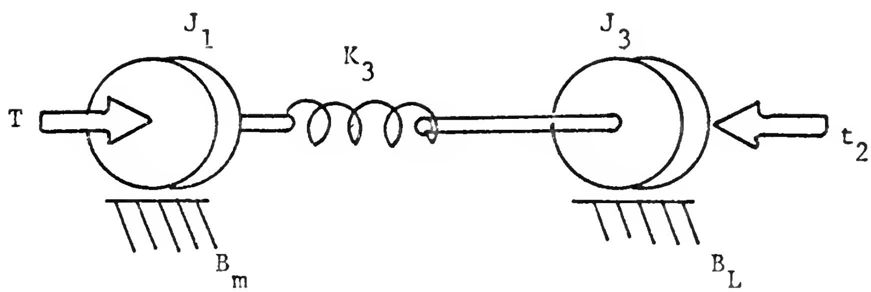
MATHEMATICAL MODELS OF BALL BEARING SCREW SYSTEM

A



Lumped Parameter Mathematical Model

B



Simplified System Model

$$B_L = \frac{22.1 \times 10^6}{4500^2} = 1.092 \text{ in lb sec.} \tag{275}$$

$$K_2 = \frac{3130 \times 10^6}{20.25 \times 10^6} = 154.5 \tag{276}$$

$$J_2 = \frac{\text{Rapson slide inertia}}{20.25 \times 10^6} =$$

$$\frac{1}{N^2} \frac{m^2}{3} = \frac{(5)(32.2)^2}{(32.2)(12)(3)} \left(\frac{1}{4500^2}\right) = \text{Negligible} \tag{277}$$

$$J_{P1} = 1.15 \times 10^{-3}(1.08^4)(1.08) = 2.02 \times 10^{-3} \tag{278}$$

$$J_{G1} = \frac{1.15 \times 10^{-3}(7.8^4)(1.08)}{(7)^2} = .292 \tag{279}$$

$$J_{P2} = \frac{1.15 \times 10^{-3}(1.67^4)(3.35)}{7^2} = .614 \times 10^{-3} \tag{280}$$

$$J_{G2} = \frac{1.15 \times 10^{-3}(7.81)(3.35)}{(32.7)^2} = .0134 \tag{281}$$

Inertia of 2 ball nuts

$$J_{BB} = \frac{1.15 \times 10^{-3}(4.25^4 - 2.587^4)(27)}{32.7^2} = 8.16 \times 10^{-3} \tag{282}$$

Inertia of hydraulic pump $J_m = .225$ (283)

The sum of these inerties is

$$\begin{aligned} J_1 &= .002 \\ & .292 \\ & .0006 \\ & .0134 \\ & .0081 \\ & \underline{.225} \\ J_1 &= .5411 \text{ in lb sec}^2 \end{aligned} \tag{284}$$

Because J_2 has been shown to be negligible K_1 and K_2 may be combined into an equivalent K_3

$$K_3 = \frac{K_1 K_2}{K_1+K_2} = \frac{(.627)(154.5)}{627 + 154.5} = 124 \ \frac{\text{in lbs}}{\text{rad}} \tag{285}$$

Combining these spring constants into an equivalent single spring allows construction of a new model shown in Figure (XVIIIB). The equations for this model are as follows

$$T = J_1\theta_1'' + B_m\theta_1' + K_3(\theta_1 - \theta_3) \tag{286}$$

$$K_3 = (\theta_1 - \theta_3) = J_3\theta_3'' + B_L\theta_3' + t_2 \text{ where } t_2 = .404\theta_3 \tag{287}$$

$$K = .404$$

$$\theta_1 = \frac{J_3}{K_3}\theta_3'' + \frac{B_L}{K_3}\theta_3' + (\frac{K + K_3}{K_3})\theta_3 \tag{288}$$

then

$$T = \frac{J_1 J_3}{K_3}\theta_3'''' + \frac{J_1 B_L}{K_3}\theta_3''' + \frac{J_1(K + K_3)}{K_3}\theta_3''$$

$$+ \frac{B_m J_3}{K_3}\theta_3''' + \frac{B_m B_L}{K_3}\theta_3'' + \frac{B_m(K + K_3)}{K_3}\theta_3'$$

$$+ J_3\theta_3'' + B_1\theta_3' + K\theta_3 \tag{289}$$

This equation may now be written in terms of θ_R the rudder angle.

$$T = \frac{J_1 J_3}{K_3}N\theta_R'''' + (\frac{J_1 B_L + J_3 B_M}{K_3})N\theta_R''' + \left\{[\frac{J_1(K+K_3)+B_m B_L}{K_3}]+J_3\right\}N\theta_R''$$

$$N\left\{\frac{B_m(K + K_3)}{K_3} + B_L\right\}\theta_R' + KN\theta_R \tag{290}$$

Now add the hydraulic motor equation to the system.

$$T = \frac{C_p D_m}{C_L}x - \frac{D_m^2}{C_L}\dot{\theta}_m = \frac{C_p D_m x}{C_L} - \frac{D_m^2}{C_L}\dot{\theta}_1 \tag{291}$$

Now substitute in the relation between θ_1 and θ_3

$$T = \frac{C_p D_m}{C_L} x - \frac{D_m^2 J_3}{C_L K_3} \theta_3''' - \frac{D_m^2 B_L}{C_L K_3} \theta_3'' + \frac{D_m^2 (K + K_3)}{C_L K_3} \theta_3' \tag{292}$$

Now convert θ_3 to θ_R in this expression, equate it to the system T equation, and combine terms.

$$\frac{C_p D_m}{C_L} x = \frac{J_1 J_3 N}{K_3} \theta_R'''' + [\frac{J_1 B_L + J_3 B_m}{K_3} + \frac{D_m^2 J_3}{C_L K_3}] N \theta_R'''$$
$$+ [\frac{J_1 (K + K_3) + B_m B_L}{K_3} + \frac{D_m^2 B_L}{C_L K_3} + J_3] N \theta_R''$$
$$+ [\frac{B_m (K + K_3)}{K_3} + \frac{D_m^2 (K + K_3)}{C_L K_3} + B_L] N \theta_R' + K N \theta_R \tag{293}$$

Then in terms of x and θ_R,

$$x = \frac{J_1 J_3 N C_L}{C_p D_m} \theta_R'''' + [\frac{J_1 B_L + J_3 B_m}{K_3} + \frac{D_m^2 J_3}{C_L K_3}] \frac{N C_L}{C_p D_m} \theta_R'''$$
$$+ [\frac{J_1 (K + K_3) + B_m B_L}{K_3} + \frac{D_m^2 B_L}{C_L K_3} + J_3] \frac{N C_L}{C_p D_m} \theta_R''$$
$$+ [\frac{B_m (K + K_3)}{K_3} + \frac{D_m^2 (K + K_3)}{C_L K_3} + B_L] \frac{N C_L}{C_p D_m} \theta_R' + \frac{K N C_L}{C_p D_m} \theta \tag{294}$$

This can be written in the shortened form

$$x = A\theta_R'''' + B\theta_R''' + C\theta_R'' + D\theta_R' + E\theta \tag{295}$$

Now consider position feedback.

$$x = C_1 \varepsilon = C_1 (\theta_R^* - \theta_R) \tag{296}$$

Using operator notation where $s = \frac{d}{dt}$, the system equation becomes

$$C_1 = [A\, s^4 + B\, s^3 + C\, s^2 + D\, s + E](\theta_R^* - \theta) \tag{297}$$

$$\frac{\varepsilon}{\theta_R^*} = \frac{[A\, s^4 + B\, s^3 + C\, s^2 + D\, s + E]}{[A\, s^4 + B\, s^3 + C\, s^2 + D\, s + E + C_1]} \tag{298}$$

Steady state error = 1/4° in 35°

$$\frac{\varepsilon}{\theta_R^*} = \frac{.25°}{35.00} = .00714 = \frac{E}{E+C_1} \tag{299}$$

$$C_1 = \frac{E(1 - .00714)}{.00714} \tag{300}$$

With the value of C_1 determined, the transfer function of the system may be written.

$$\frac{\theta_R}{\theta_R^*} = \frac{C_1}{A\ s^4 + B\ s^2 + C\ s^2 + D\ s + E + C_1} \tag{301}$$

Now solve for the constants, plot on a polar plot, determine M and relate this to an equivalent damping constant ξ. In this way the characteristics of the oscillatory behavior of the system can be determined.

Evaluation of Constants

$$A = J_1 J_3\ N \frac{C_L}{C_p D_m} = (.5411)(.038)\ \frac{4500}{79500} = (.541)(.038)(.0566)$$
$$= 1.16 \times 10^{-3} \tag{302}$$

$$B = \left[\frac{J_1 B_L + J_3 B_m}{K_3} + \frac{D_m^2 J_3}{C_L K_3}\right] \frac{NC_L}{C_p D_m} = \left[\frac{(.51)(1.092)+(.038)(.216}{124}\right.$$
$$\left. + \frac{(.772^2)(.038)}{(.005)(124)}\right] (.0566)$$

$$= \left[\frac{.567}{124} + .0365\right].0566 = (.041)(.0566) = 2.32 \times 10^{-3} \tag{303}$$

$$C = \left[\frac{J_1(K + K_3)+B_m B_L}{K_3} + \frac{D_m^2 B_L}{C_L K_3} + J_3\right] \frac{NC_L}{C_p D_m} = \frac{.511(.404+124)+(.216)(1.092)}{124}$$
$$+ \frac{(.772^2)(1.092)}{(.005)(124)} + .038]\ .0566$$

$$= [.00604 + 1.05 + .038]\ .0566 = .0619 \tag{304}$$

$$D = [\frac{B_m(K + K_3)}{K_3} + \frac{D_m^2(K + K_3)}{C_L \cdot K_3} + B_L] \frac{NC_L}{C_p D_m} = [\frac{(.216)(124)}{124}$$

$$+ \frac{(.772^2)(124)}{(.005)(124)} + 1.092].0566$$

$$= [.216 + 119 + 1.092]\ .0566 = 6.92 \qquad (305)$$

$$E = \frac{KNC_L}{C_p D_m} = (.404)(.0566) = .0229 \qquad (306)$$

$$C_1 = \frac{.0299(.99286)}{.00714} = 3.18 \qquad (307)$$

These can be substituted into the transfer function to obtain the following form.

$$\frac{\theta_R(j\omega)}{\theta_R^*(j\omega)} = \frac{3.18}{.00116\omega^4 + .00232(-j\omega^3) + .0619(-\omega^2) + 6.92j\omega + 3.203}$$

$$= \frac{3.18}{(.00116\omega^4 - .0619\omega^2 + 3.203 + j(-.00232\omega^3 + 6.92\omega)} \qquad (308)$$

The transfer function may be written in the form

$$\frac{\theta_R}{\theta_R^*} = \frac{X}{Y + jZ} \qquad (309)$$

The closed loop frequency response may be determined from plotting the open loop response[27]. By definition the open loop response $G(j\omega)$ is related to the system transfer function by the following equation.

$$\frac{\theta_R(j\omega)}{\theta_R^*(j\omega)} = \frac{G(j\omega)}{1 + G(j\omega)} = \text{transfer function} \qquad (310)$$

$$\frac{G}{1+G} = \frac{X}{Y + jZ}$$

$$G = (\frac{X}{Y + jZ})(1+G)$$

$$G(1 - \frac{X}{Y + jZ}) = \frac{X}{Y + jZ}$$

$$G = \frac{X}{Y + jZ} (\frac{Y + jZ}{Y - X + jZ}) = \frac{X}{Y - X + jZ} \qquad (311)$$

Now substitute in the numerical values from equation (308)

$$G = \frac{3.18}{(.00116\omega^4 - .0619\omega^2 + 3.203 - 3.18) + j(-.00232\omega^3 + 6.92\omega)}$$

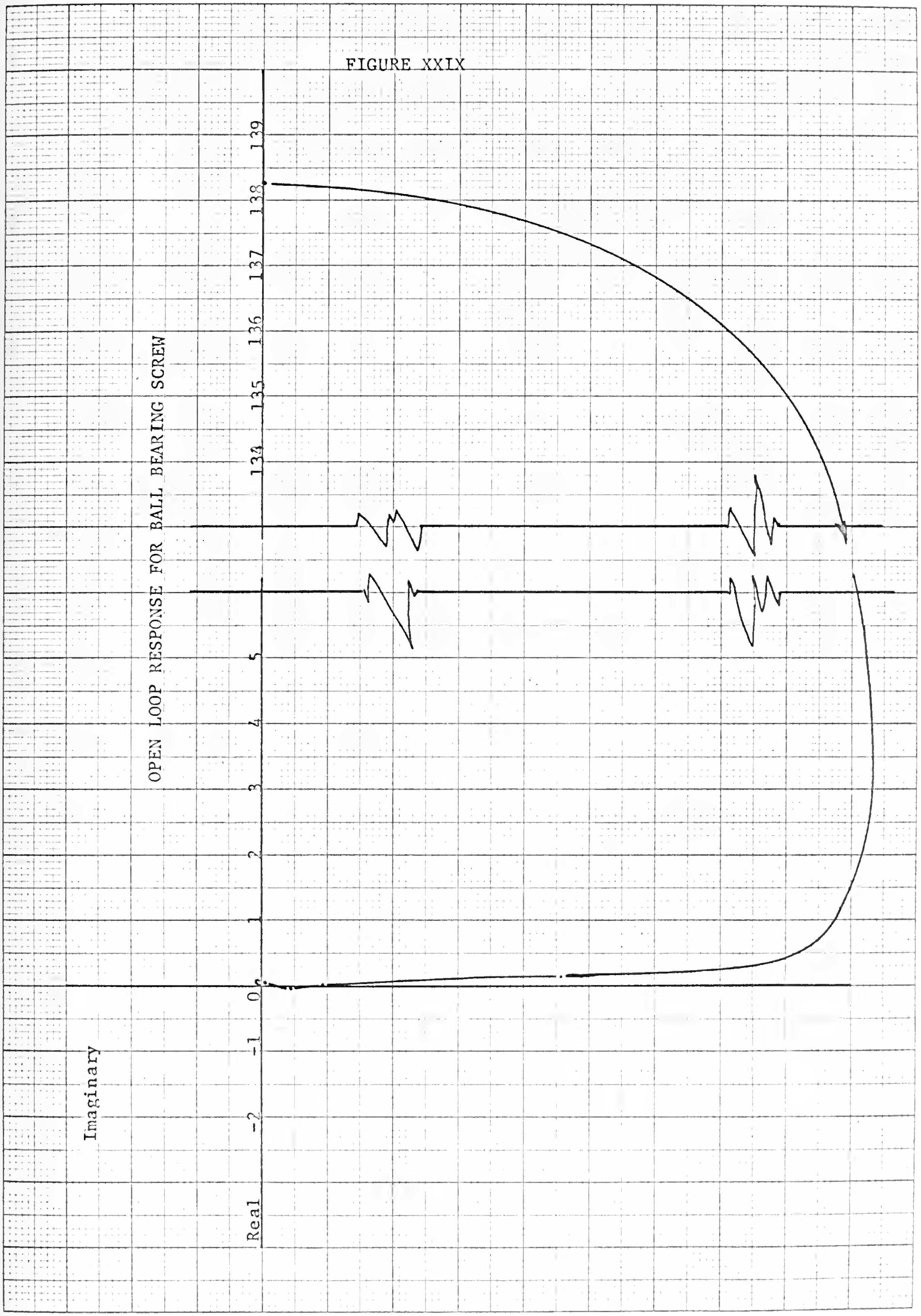
$$= \frac{3.18[(.00116\omega^4 - .0617\omega^2 + .023) - j(-.00232\omega^3 + 6.92\omega)]}{(.00116\omega^4 - .0619\omega^2 + .023)^2 + (-.00232\omega^3 + 6.92\omega)^2} \qquad (312)$$

In order to plot G in the complex plane, the following indicator points are calculated.

ω	Real $G(j\omega)$	Imaginary $G(j\omega)$
$\omega \to 0$	$\frac{(3.18)(.023)}{.023^2} = 138.2$	0
$\omega = .1$	$\frac{(3.18)(.023)}{(.023)^2 + (.692)^2} = +.152$	$-\frac{(3.18)(.692)}{.69^2} = -4.6$
$\omega = .5$	$\frac{3.18(-.0151+.023)}{(-.0151+.023)^2 + (3.46)^2} = +.0021$	$\frac{-(3.18)\ (3.46)}{(-.0511+.023)^2 + (3.46)^2} = -.92$
$\omega = 1$	$\frac{(3.18)(-.0376)}{(-.0376)^2 + (6.92)^2} = +.0021$	$\frac{-(3.18)(6.92)}{(.0376)^2 + (6.92)^2} = -.46$
$\omega = 10$	$\frac{3.18(11.62 - 6.17)}{(11.62-6.17)^2 + (-2.32+69.2)} = +.0039$	$\frac{-(3.18)(66.8)}{4509} = -.044$
$\omega \to \infty$	$\frac{\omega^4}{\omega^8} \to +0$	$\frac{-(-\omega^3)}{\omega^8} \to +0$

These values are plotted in Figure (XXIX). It can be seen from this plot that the behavior of this system closely approximates that of a first order for values of ω greater than .1. In fact it may be approximated by a first order system whose time constant is the reciprocal of ω at the point where $G(j\omega) = 1$[27]. In this case for G = 1,

FIGURE XXIX



OPEN LOOP RESPONSE FOR BALL BEARING SCREW

$$\omega = .45 \text{ (approx)} \tag{313}$$

$$\tau = \frac{1}{.45} = 2.22 \tag{314}$$

The response of the rudder to a large rudder angle (approximated by a step input) is given by the following equation.

$$\theta_R = \theta_R^*(1 - e^{-\frac{t}{2.22}}) \text{ where } t = \text{time in seconds} \tag{315}$$

The above analysis is based upon the presumption that the system operates in the proportional control mode. Actually the hydraulic motor staturates (reaches maximum pumping rate) quickly and so the possibility of a limit cycle instability must be considered. The non linearity is the same as the one in the harmonic gear analysis and so the values of $-\frac{1}{K_{eq}}$ are the same as before going from -1 to $-\infty$ along the negative real axis. A first order system could not possibly intersect the trace of these values in a polar plot[28]. Therefore a limit cycle is not possible and the system is stable.

3.3.3.4 Discussion

In summary, the system has been shown to be heavily damped and as a result its response is that of a simple time lag. Simple position feedback has been shown to be satisfactory, and the system is stable in all modes. The normal cause of limit cycle instability in ball bearing screws is the backlash of the device, but this has been eliminated in this system by the method of attachment of the ball nuts.

The use of ball bearing screws for steering engines in this torque range is shown to be heavily dependent on the lifetime obtainable for the device. In order to go to higher torques, the nominal radius R

$$\omega = .45 \text{ (approx)} \qquad (313)$$

$$T = \frac{1}{.45} = 2.22 \qquad (314)$$

The response of the rudder to a large rudder angle (approximated by a step input) is given by the following equation.

$$\theta_R = \theta_c\left(1 - e^{-\frac{t}{2.22}}\right) \quad \text{where } t = \text{time in seconds} \qquad (315)$$

The above analysis is based upon the assumption that the system operates in the proportional control mode. Actually the hydraulic motor saturates (reaches maximum pumping rate) quickly and so the possibility of a limit cycle instability must be considered. The non-linearity is the same as the one in the harmonic gear analysis and the values of $-\frac{1}{K_{eq}}$ are the same as before going from -1 to $-\infty$ along the negative real axis. A first order system could not possibly intersect the trace of these values in a polar plot [illegible]. Therefore a limit cycle is not possible and the system is stable.

3.3.7.4 Discussion

In summary, the system has been shown to be basically linear and as a result its response is that of a simple time lag. Simple position feedback has been shown to be satisfactory and the system is stable in all modes. The normal cause of limit cycle instability in ball bearing screws is the backlash of the device, but this has been eliminated in this system by the method of attachment of the ball screw.

The use of ball bearing screws for steering gear [illegible] range is shown to be [illegible] dependent upon the life [illegible] the device. In order to [illegible] would have to be increased and perhaps another screw fitted to operate in parallel with the first one. However, these arrangements have their disadvantages and the increase in torque capacity would not be great. Consequently it can be concluded that in the light of present manufacturing limitations and the requirement of designing for a twenty year lifetime, the medium high torque range is the upper limit for the use of ball bearing screws. It is to be noted in passing that if the lifetime requirements were only that sufficient to ensure trouble free operation during the normal overhaul cycle of from three to four years, these units could be adapted to very high torque installations. The feasibility of this scheme would necessarily depend upon a careful cost analysis.

3.3.4 Hydrostatic Bearing Devices

Hydrostatic bearings are a type of fluid film bearing in which the lubricant is pumped under pressure into the clearance space between the bearing surfaces. The formation of the fluid film therefore does not depend on the relative speed of the bearing surfaces. Rather it is a function of the rates of supply and leakage of the lubricant. There are several effective methods now in practice of controlling and regulating the flows in and out of the bearing depending on the requirements of the application. The analytic equations for several geometries have been solved including the radial thrust bearing, and a wide variety of these bearings are available commercially. The bearing enjoys the same high efficiency under load as other fluid film bearings, but the pump which necessarily runs continuously absorbs power when there is no load on the bearing. However, with proper design, this can be kept quite small.

would have to be increased and (or) another screw fitted to operate in parallel with the first one. However, these arrangements have their disadvantages and the increase in torque capacity would not be great. Consequently it can be concluded that in light of present manufacturing limitations and the requirement of design for a twenty year lifetime, the reduction of torque range is the same as that for the use of ball bearing screws. It is to be noted in passing that if the lifetime requirements were only that sufficient to ensure trouble free operation during the normal overhaul cycle of from three to four years, these unit could be adapted to very high torque installations. The feasibility of this scheme would necessarily depend upon a careful cost analysis.

3.3.4 Hydrostatic Bearing Designs

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The concept of the hydrostatic bearing can probably be adapted to a steering engine design without too much difficulty. It is envisioned that a screw could be threaded into a nut with the threads kept out of contact by the film of hydrostatically pressurized lubricant. The idea is similar to that of a ball bearing screw except that the balls be replaced by an oil film. The efficiency would be markedly better than a conventional screw although the dimensions would have to be somewhat larger. This is because pumps and lubricant technology limit pressure to the 5000 psi range, while hardened steel gear teeth may support surface pressures well over 100,000 psi. However it must be remembered that generally only a small percentage of the total tooth face area is actually in contact in a conventional gear. It appears quite favorable in comparison with the ball bearing screw, because it does not have any fatigue lifetime restrictions. It would probably be comparable to the ball screw efficiency, but it might easily be smaller and lighter. This linear hydrostatic actuator could then be used in conjunction with some standard mechanism, probably a rapson slide, to convert its output to angular rotation. If a hydraulic pump-motor drive transmission were used, there would be a readily available source of pressurized oil for the bearing.

The idea could be extended to a worm and wheel application. It would require some sort of supply manifold but this is not insurmountable. However this design would be expected to have low torque capacity. This is because only a small percentage of the wheel periphery is available for contact area, and then this area can only be loaded to 5000 psi. For this reason, the hydrostatic worm does not appear to have an application in a steering engine.

The concept of the hydrostatic bearing can be [illegible] to a screw-nut [illegible] without too much difficulty. It is envisioned that a screw could be threaded into a nut with the threads kept out of contact by a film of hydrostatically pressurized fluid. The idea is similar to that of a ball bearing screw except the balls would be replaced by an oil film. The efficiency would be much better than a conventional screw although the dimensions would have to be somewhat larger. This is because pumps and lubricant technology limit pressure to the 5000 psi range, while hardened steel gear teeth can support surface pressures well over [illegible] psi. However, it must be remembered that generally only a small percentage of the total surface area is actually in contact in a conventional gear. It [illegible] favorable in comparison to the ball bearing screw, because it would not have any fatigue life limitations. It would probably be comparable to the ball screw efficiency, but it would be easily made smaller and lighter. This linear hydrostatic actuator could then be used in conjunction with some standard mechanism such as a [illegible] slide, or convert its output to angular rotation. If a hydraulic pump-motor drive transmission were used, there would be a readily available source of pressure fluid for the bearing.

The idea could be extended to a worm and worm gear application. It would require a [illegible] manifold but this is not insurmountable. However this design would be expected to have low torque capacity. This is because only a small percentage of the wheel teeth area is available for contact at any time. This area can only be [illegible]. For this reason, the hydrostatic worm does not appear to have an application in a [illegible] engine.

The developmental work required to make this concept a reality appears to be modest. As previously mentioned, the analytic equation for radial geometry has been solved and several styles of radial thrust bearings are presently performing satisfactorily in a wide assortment of commercial applications. What is required is an analysis to adapt the two dimensional radial solution to a three dimensional continuous helix. Then a working model would have to be constructed and tested to verify the analytic results. It is clear that the solution of this problem is well within the capabilities of present technology, and evolution of a practical design is certainly an avenue worthy of exploration in the future. It is regretted that insufficient time was available during the writing of this paper to do so.

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CHAPTER IV

DISCUSSION OF RESULTS

The direct acting electro-magnetic devices were shown to be non competitive on the basis of an examination of their fundamental characteristics. They operate most efficiently as high speed low torque devices which is the diametric opposite of the requirements of a steering engine.

The electro-hydraulic machines were considered next. Whenever it was required to convert a linear actuator's output to rotary motion of the rudder stock, the rapson slide was shown to result in the lightest system because of its shorter length of travel. The weights for rapson slides of various radii are estimated in Appendix V. These weights are probably conservative because no consideration was taken of the weights of bolts, foundations, and other assemblage details nor of the weight of miscellaneous ancillary equipment. Further if a different design arrangement were chosen, it is possible that the results would be somewhat at variance with Appendix V. However, it is felt that these calculations do provide a good insight into the relative orders of magnitude involved.

The weight equations for the hydraulic piston and cylinder were shown to have an optimum pressure with respect to weight determined by the parameter $p/f\sigma_y$. The result of these calculations are shown in Figures (VI) and (IX). It is noted that cylinder dimensions go down dramatically as pressure is increased if reasonably high yield steel is used. Although high pressures have caused difficulty in the past due to formation of local shock waves, the piston and cylinder is expected to be reasonably free from this condition because of its simplicity and

CHAPTER IV

DISCUSSION OF RESULTS

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The electro-hydraulic [illegible] were considered [illegible]. When it was required to convert a linear actuator's output to rotary motion of the rudder stock, the rapson slide was shown to result in the lightest system because of its shorter length of travel. The weights for rapson slides of various radii are estimated in Appendix V. The [illegible] are probably conservative because no consideration was taken of the weights of bolts, foundations, [illegible] sealing details nor of the weight of miscellaneous auxiliary equipment. Further if a different design arrangement was chosen, it is possible that the results would be somewhat at variance with [illegible]. However, it is felt that these calculations do provide a good insight into the relative orders of magnitude involved.

The weight equations for the hydraulic piston and cylinder were shown to have an optimum pressure with respect to weight [illegible] by the parameter [illegible]. The results of these calculations are shown in Figures (11) and (12). It is [illegible] that cylinder weight goes down dramatically as pressure is increased if reasonably high yield steel is used. Although high pressure [illegible] caused difficulty in the past due to formation of local shock waves, the piston and cylinder are expected to be reasonably free from this condition because [illegible] the absence of control values. The weights have been calculated using the suggested procedure and are reasonably accurate for a bare piston and cylinder. Not considered were the weights of seals, foundations and piping fittings. The summary of the weights of a hydraulic piston and cylinder with a rapson slide for various values of R is as follows:

	R=19	R=25	R=30	R=40	R=50
Rapson Slide	1791	1743	1672	1627	1637
Piston & Cylinder (optimum $\bar{p}$ for $f\sigma_y$ = 17,500 psi)	986	864	840	790	768
Hydraulic Pump	310	310	310	310	310
Drive Motor	360	360	360	360	360
	3,447lbs	3,277lbs	3,182lbs	3,087lbs	3,075lbs

For a value of $f\sigma_y$ = 35,000 psi, pressure was limited to 5,000 psi by practical considerations. Weights for the piston and cylinder are as follows:

R=19	R=25	R=30	R=40	R=50
497 lbs	466 lbs	437 lbs	386 lbs	384 lbs

It is noted that total weight decreases with increasing R which at first implies the desirability of large values of R. However, space is an important consideration particularly on a volume limited ship, and therefore large R is not usually desirable. In this regard it is noted that the amount of weight that can be saved by increasing R is only a small percentage which may not be worthwhile to obtain.

A rotary vane actuator was shown to be applicable to the full torque range of steering engines. It has the attraction of inherent simplicity with fewer moving parts than any other system. The leakage rate was found to exert a minor influence on the overall design. Of

far more importance is the weight of the cylindrical shell and the end plates which account for the major part of the overall weight of the system. An approach to optimizing this weight similar to the method used for the piston and cylinder is suggested. The weights were calculated to be as follows:

	p = 3,000 psi	p = 5,000 psi
Rotary Vane Actuator	7225	7479 lbs
Hydraulic Pump	110	110
A.C. Drive Motor	520	492
	7,745	8,081

This weight may be over estimated since the optimization procedure suggested above could reduce it somewhat. Although the weights of bearings, seals, and fastenings, which are considerable, were not taken into account, they are offset by weight saving refinements normally incorporated in the design such as rib and ring stiffened end plates and cylindrical shell.

Various gear reduction drives were surveyed and it was determined that the harmonic gear is the most promising of these. The preliminary investigation of this gear disclosed that deflection of the flexspline by use of a radial arrangement of either hydraulic cylinders or electrical solenoids was not feasible primarily because a power loss would release the rudder to swing free. A bell shaped flexspline was chosen because of its higher load capacity and a design was worked out. The weights for this design are as follows:

for [illegible] is [illegible] plates [illegible] the pilot [illegible] system. [illegible] for the [illegible] cylinder is suggested [illegible] to be as follows:

	[illegible]	[illegible]
Rotary [illegible] Actuator	72[illegible]	[illegible]
Hydraulic Pump	11	[illegible]
A.C. Drive Motor	[illegible]	[illegible]
	7,[illegible]	[illegible]

This weight may be over estimated [illegible] procedure suggested above could reduce it somewhat. [illegible] weights of bearings, [illegible], and fastenings, which are considerable, were not taken into account, they are offset by weight [illegible] incorporated in the [illegible] and cylindrical [illegible].

Various gear reduction drives were [illegible] that the [illegible] gear is the most promising of [illegible] investigation of [illegible] by use of a radial arrangement of [illegible] electrical solenoids was not [illegible] primarily because [illegible] loss would release the rudder to swing free. A ball [illegible] of its higher load capacity [illegible] a design [illegible] for this design are as follows:

Harmonic Gear	6,140 lbs
Drive Gear Train	34 lbs
A.C. Motor plus Hydraulic Transmission	862 lbs
Drive Motor support platform	100 lbs
	7,136 lbs

These weights are probably very accurate as they are based on empirical data from the manufacturer. As a result they probably suffer in comparison with other systems whose weights are not nearly so accurately determined. The lifetime of the ball bearing races is an area of concern. Although it was shown that a satisfactory life was obtainable in this case, this may not be true for very much higher torques. A simple proportional control system was shown to be acceptable although some oscillation in the drive end may occur. The great compliance of the gear produces such a low natural frequency for the rudder that the possibility of vibrations of it and the hull become a real possibility. This requires that a careful vibration investigation of each particular rudder and ship system be performed prior to installation of this device.

The use of ball bearing screws was investigated, and it was determined that present manufacturing limitations restrict its use to the low and medium high torque ranges. A design which included a rapson slide was worked out which demonstrated that satisfaction of the lifetime criteria is the major consideration of the design calculation. Simple proportional control was shown to be acceptable. Weights were calculated for the system as follows.

[illegible]

[illegible]

[illegible]

[illegible]

These weights are [illegible] very accurate [illegible] based on empirical data from [illegible] in comparison with [illegible] accurately determined. The [illegible] area of [illegible] Although it [illegible] shown that [illegible] obtainable in this case, this may not be true for [illegible] torques. [illegible] acceptable although [illegible] oscillation in the drive [illegible] great complaints of [illegible] produces such [illegible] for the rudder that the possibility of vibrations [illegible] a real possibility. This [illegible] of each particular [illegible] installation of this valve.

The use of all [illegible] determined that present [illegible] the low [illegible] rapson slide was [illegible] the lifting criteria [illegible] calculation. [illegible] control was [illegible] Weights were calculated for the system [illegible] follows.

Rapson Slide	1539
Ball Bearing Screw	870
Drive Gear Train	499
Hydraulic Transmission etc.	370
A.C. Drive Motor	410
	3688 lbs

These weights are probably reasonably accurate and are quite valid for relative comparisons with the other systems.

Finally the idea of a hydrostatic bearing device in the form of a linear screw actuator was proposed. It was shown to be desirable as a way of producing an efficient electro mechanical device which would be free of anti-friction bearing lifetime considerations. Development of this design appears to be technically feasible and the results promising. It could result in a smaller design than the ball bearing screw.

Some insight into the relative efficiencies of the proposals can be gained by looking at the required horsepower of each drive motor.

	Required Drive Power	Installed Motor
Piston & Cylinder, Rapson Slide	35.0 H.P.	25 H.P.
Rotary Vane Actuator	40.1 H.P.	30 H.P.
Harmonic Gear	56 H.P.	40 H.P.
Ball Bearing Screw	40.1 H.P.	30 H.P.

Each drive motor is assumed to have an overload capacity of 50% which is available for the intermittant peak power requirements of a steering engine.

Differences in the above figures are probably due to the differences in the machines themselves, because all of them are driven by the

same hydraulic pump running at the same speed of 1750 rpm. Therefore they can form the basis of a valid efficiency comparison.

CHAPTER V

CONCLUSIONS

The major conclusion that emerges from this investigation is that the hydraulic piston and cylinder working through a rapson slide is the best steering engine. It has the lowest overall weight and the highest efficiency of all the systems considered. The weight of the piston and cylinder can be reduced drastically by using the suggested optimization procedure to employ higher hydraulic pressures. The rapson slide is superior to the tiller and linkage, and rack and pinion because of its shorter length of travel.

The rotary vane actuator is attractive because of its inherent simplicity, and it is applicable to the full range of rudder torques. However, it is the heaviest of the systems considered. Some reduction in weight can be expected if an optimization procedure is developed.

The harmonic gear is the most attractive of the possible gear reduction drives. The bell shaped ball bearing actuated flexspline configuration is considered to be the most suitable for this application. Actuation of the flexspline by radially located hydraulic pistons or electrical solenoids is not feasible. The system is relatively heavy, and it has the lowest efficiency of the group. The high compliance of the gear makes the natural frequency of the rudder undesirably low which introduces the possibility of rudder and hull vibrations.

The use of ball bearing screws is feasible but limited by manufacturing considerations from use in the high torque range. Ball bearing life is the critical design factor. The system is quite light in weight and has good efficiency.

Development of a hydrostatic linear screw actuator is technically feasible and desirable.

Direct acting electro-magnetic devices are not feasible for use as steering engines.

CHAPTER VI

RECOMMENDATIONS

It is recommended that the hydraulic piston and cylinder acting through a rapson slide be considered first for a steering engine. The suggested optimization process should be used to increase pressures and lower the piston and cylinder dimensions and weight. If extreme ruggedness and simplicity are required for some reason, then the rotary vane actuator should be considered. In this event a weight optimization study should be undertaken.

It is recommended that some effort be expended on the analytic and experimental development of hydrostatic bearing devices to adapt them to use in steering engines.

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APPENDIX

APPENDIX I

CALCULATION OF CHARACTERISTICS OF REPRESENTATIVE RUDDER

Calculation Of Physical Inertias

Hub casting inertia

$$I_H = \frac{\rho\pi h}{10(r_2-r_1)}\,[R_2^5-R_1^5-(r_2^5-r_1^5)] \quad \text{where } R_{1,2} = \text{outer radii}$$

$$r_{1,2} = \text{inner radii}$$

$$= \frac{\rho\pi 85}{(10)(7)}\,[35.6\text{x}10^5-5.39\text{x}10^5+.59\text{x}10^5-10.5\text{x}10^5]$$

$$= \frac{(.283)(\ \ )(85)(25.7\text{x}10^5)}{g\ 70} = \frac{2.77 \times 10^6}{g} \qquad (316)$$

Rudder stock

For ease of calculation the stock is divided into four parts at each abrupt change in diameter. These are numbered from the bottom up.

$$I_1 = \frac{\rho\pi h}{10}\,[\frac{R_2^5-R_1^5}{R_2-R_1} - 5r^4] \quad \text{where} \quad R_1 = 9''$$

$$R_2 = 16''$$

$$r_4 = 5''$$

$$r = 625$$

$$= \frac{\rho\pi 85}{10}\,[\frac{9.9 \times 10^5}{7} - 5(625)]$$

$$1.43 \times 10^5 - 3125$$

$$= (\frac{.283}{g})(\pi)(85)(1.40 \times 10^4) = \frac{1.058 \times 10^6}{g} \qquad (317)$$

$$I_2 = \frac{\rho\pi h}{2}\,[R^4 - r^4]$$

$$= \frac{(.283)(\pi)(53)}{g^2}\,[65600-625] = \frac{1.53 \times 10^6}{g}$$

$$I_3 = \frac{\rho\pi h}{10}\left[\frac{R_2^5-R_1^5}{R_2-R_1} - 5r^4\right]$$

$$= \left(\frac{.283}{g}\right)\left(\frac{\pi 62}{10}\right)(1.24 \times 10^5) = \frac{.684 \times 10^6}{g} \qquad (319)$$

$$I_4 = \frac{\rho\pi h}{2}\left[R^4 - r^4\right] \qquad (320)$$

$$= \frac{(.283)(\pi)(22)}{g^2}[4096-625] = \frac{3.4 \times 10^3}{g}$$

For the inertias of Shell plating and stifferners

$$I = \frac{m\ell^2}{3}$$

Long'l stiffeners aft. of stock

$$I = \frac{2420}{g}\ \frac{(96)^2}{3} = \frac{7.44 \times 10^6}{g} \qquad (321)$$

Long'l stiffeners fwd. of stock

$$\text{(A)} \quad I = \frac{740}{g}(34)^2 = \frac{.855 \times 10^6}{g} \qquad (322)$$

$$\text{(B)} \quad I = \frac{724}{g}\ \frac{24^2}{3} = \frac{.014 \times 10^6}{g} \qquad (323)$$

Vertical Stiffeners.

The vertical stiffeners are numbered from the stock aft. as 1, 2, 3, 4 and from the stock forward as 01 and 02.

$$\text{(1)} \quad I = m\ell^2 = \frac{363}{g}13^2 = \frac{61,400}{g} \qquad (324)$$

$$\text{(2)} \quad I = \frac{775}{g}(39)^2 = \frac{1,180.00}{g} \qquad (325)$$

$$\text{(3)} \quad I = \frac{554}{g}(64)^2 = \frac{2,270,000}{g} \qquad (326)$$

$$\text{(4)} \quad I = \frac{218}{g}(77)^2 = \frac{1,290,000}{g} \qquad (327)$$

$$(01)\ I = \frac{246}{g}(30)^2 = \frac{222,000}{g} \tag{328}$$

$$(02)\ I = \frac{224\ 9^2}{g} = \frac{18,100}{g} \tag{329}$$

Total vertical stiffeners

$$\frac{5.041}{g} \times 10^6 \tag{330}$$

Side Plating

Aft. of Stock

$$I = \frac{m\ell^2}{3} = \left(\frac{6,580}{g}\right)\frac{(102)^2}{3} = \frac{22.8 \times 10^6}{g} \tag{331}$$

Fwd. of stock

$$I = \frac{m\ell^2_{max}}{6} = \left(\frac{2,250}{g}\right)\left(\frac{53^2}{6}\right) = \frac{1.052 \times 10^6}{g} \tag{332}$$

Total Rudder Mechanical Inertia

$$\begin{array}{ll}
\text{Horiz. stiff} \left\{ \begin{array}{l} \\ \\ \\ \end{array} \right. & \begin{array}{l} 7.44 \times \frac{10^6}{g} \\ .855 \quad '' \\ .014 \quad '' \end{array} \\
\text{Vert. stiff} & 5.041 \quad '' \\
\text{Plating} \left\{ \begin{array}{l} \\ \\ \end{array} \right. & \begin{array}{l} 22.80 \quad '' \\ 1.052 \quad '' \end{array} \\
\text{Hub casting} & \underline{2.77\qquad\quad} \\
\text{Total} & 39.972 \times \frac{10^6}{g}
\end{array} \tag{333}$$

Rudder Stock Inertia

$$\begin{array}{ll}
I_1 & 1.058 \times \frac{10^6}{g} \\
I_2 & 1.53 \quad '' \\
I_3 & .684 \quad ''
\end{array}$$

$$I_4 \qquad \frac{.003\text{x}\ \frac{10^6}{g}}{}$$

$$\text{Total} \qquad 3.275 \times \frac{10^6}{g} \tag{334}$$

Total Inertia

$$= \frac{[39.97 + 3.27]\text{x } 10^6}{386.4} = 1.12 \times 10^5 \text{ in lbs sec}^2 \tag{335}$$

STIFFNESS OF STOCK

The same sections of the stock as were used to calculate inertia are used here.

$$\theta_{(1)} = \frac{T\ell_{(1)}}{GJ_{(1)}} = \frac{T\ell 10}{G\pi\ (\frac{R_2^5-R_1^5}{R_2-R_1} - 5r^r)} = \frac{(5\text{x}10^6)(85)(10)}{(12\text{x}10^6)(\pi)(1.40\text{x}10^5)} = 8.05\text{x}10^{-4} \text{ rad} \tag{336}$$

$$\theta_{(2)} = \frac{T\ell_{(2)}}{GJ_{(2)}} = \frac{T(53)\ell_{(2)}}{(12\text{x}10^6)(6.4975\text{x}10^{-4}} = 2.17\text{x}10^{-4} \text{ rad} \tag{337}$$

$$\theta_{(3)} = \frac{(5\text{x}10^6)(62)(10)}{(12\text{x}10^6)(\pi)(1.24\text{x}10^5)} = 6.65\text{x}10^{-4} \text{ rad} \tag{338}$$

Shaft section (4) does not contribute

$$K = \frac{T}{\theta} = \frac{5 \text{ x } 10^6}{\theta_{(1)}+\theta_{(2)}+\theta_{(3)}} = \frac{5 \text{ x } 10^6}{16.87\text{x}10^{-4}} = 3130\text{x}10^6 \ \frac{\text{in lbs}}{\text{rad}} \tag{339}$$

VIRTUAL INERTIA & VISCOUS DAMPING

Notation used is as follows[6]

M_{H_d} = Moment about a hinge due to d where d can equal $\delta, \dot{\delta}$, or $\ddot{\delta}$

C_f = Chord length

A_f = Projected area

$T_{1,2,..}$ = Theordorsen coefficients

Z = Body force at hinge in z direction with respect to body

x = fraction of chord length at which rudder stock is located

(wf) = Wake fraction

$AR = \frac{4(R_f - R_b)^2}{A_f}$ where R_f = outer f in radius

R_b = body radius

$= \frac{2\ell^2}{A_f}$ where ℓ = rudder length

U = ship speed

δ = angular displacement

$\dot{\delta}$ = angular velocity

$\ddot{\delta}$ = angular acceleration

use η = 1 Theodorsen oscillatory correction coefficient

$$M_x = M_H - Z_H \times C_f \tag{340}$$

This equation transfers the moments and forces from the leading edge to the point $x\, C_f$ from the leading edge.

Set $[1 + \frac{R_b}{R_f}]^2 = 1$

$$M_{H\ddot{\delta}} = M'_{H\ddot{\delta}}\ 1/2\ \rho\ L^5 \tag{341}$$

$$= \frac{C_f^3\ A_f\ T_3}{8\pi L^5} [1 + \frac{R_b}{R_f}]^2 \frac{1}{\sqrt{1 + \frac{1}{(AR)^2}}} \frac{1}{2}\ \delta L^5 \tag{342}$$

$$Z_{\ddot{\delta}} = \frac{C_f^2\ A_f\ T_1}{4L^4\sqrt{1 + \frac{1}{(AR)^2}}} [1 + \frac{R_b}{R_f}]^2 \frac{1}{2}\rho\ L^4 \tag{343}$$

$$M_{x_{\ddot{\delta}}} = -\frac{9C_f^3 A_f \pi\rho}{128\sqrt{1+\frac{1}{(AR)^2}}} - xC_f\left(-\frac{C_f^2 A_f \pi\rho}{8\sqrt{1+\frac{1}{(AR)^2}}}\right.$$

$$= \frac{C_f^3 A_f \pi\rho}{\sqrt{1+\frac{1}{(AR)^2}}}\left(-\frac{9}{128}+\frac{x}{8}\right) = \text{Polar "Virtual" Inertia} \quad (344)$$

$$M_{H_{\dot{\delta}}} = \frac{C_f^2 A_f}{8\pi L^4}[1-(Wf)][1+\frac{R_b}{R_f}]^2\left[\frac{(T_4)(T_{11})}{\sqrt{1+\frac{1}{(AR)^2}}} - \frac{(5.7)(T_{11})(T_{12})}{2\left(1+\frac{2}{AR}\right)}\right]\frac{1}{2}\rho L^4 U \quad (345)$$

$$Z_{\dot{\delta}} = \frac{C_f A_f}{2L^3}[1-(Wf)][1+\frac{R_b}{R_f}]^2\left[\frac{T_4}{\sqrt{1+\frac{1}{(AR)^2}}} - \frac{5.7\,T_{11}}{2\left(1+\frac{2}{(AR)}\right)}\right]\frac{1}{2}\rho L^3 U \quad (346)$$

$$M_{x_{\dot{\delta}}} = C_f^2 A_f[1-(Wf)]U\rho\left[\left(-\frac{3}{16}+\frac{x}{4}\right)\frac{1}{\sqrt{1+\frac{1}{AR^2}}}\right.$$

$$\left. + \left(-\frac{(3)(5.7)}{32}+\frac{x(5.7)(3)}{8}\right)\frac{1}{\left(1+\frac{2}{AR}\right)}\right]$$

$$= C_f^2 A_f[1-(Wf)]U\rho\left[\frac{\pi}{16}\frac{(4x-3)}{\sqrt{1+\frac{1}{AR^2}}} + \frac{(57)(3)}{8}\frac{\left(x-\frac{1}{4}\right)}{\left(1+\frac{2}{AR}\right)}\right] \quad (347)$$

DAMPING = damping coefficients

Now substitute the appropriate values into the above equations to obtain the numerical values for inertia and damping.

$$M_{\dot{\delta}} = C_f^2 A_f(1.10)U\rho\left[\frac{\pi}{16}\frac{(4x-3)}{\sqrt{1+\frac{1}{AR^2}}} + \frac{(57)(3)}{8}\frac{\left(x-\frac{1}{4}\right)}{\left(1+\frac{2}{AR}\right)}\right.$$

$$= (10.87)^2(196.6)(1.10)(27)(1.689)(1.99)\left[\frac{\pi}{16}\;\frac{[(4)(.219)-31}{1.045}\right.$$

$$\left. + \frac{(57)(3)}{8}\;\frac{(.219\;-\;.25)}{(1 + \frac{2}{3.34})}\right]$$

$$= 1.841\text{x}10^4 = 1.84\text{x}10^6 \text{ ft lb sec}$$

$$= 1.84\text{x}10^6\text{x}12\;\frac{\text{in}}{\text{ft}} = 22.1\text{x}10^6 \text{ in lb sec} \tag{348}$$

$$M_{\ddot{\delta}} = \frac{C_f^3 A_f \;\pi\rho}{\sqrt{1 + \frac{1}{AR^2}}}\;\left(-\,\frac{9}{128} + \frac{x}{8}\right)$$

$$= \frac{[(10.87)^3(196.6x)(3.14)(1.99)]}{\sqrt{1 + \frac{1}{3.342}}}\;\left(-\,\frac{\theta}{128} + \frac{.219}{8}\right)$$

$$= 54{,}800 \text{ ft lb sec}^2$$

$$M_{\ddot{\delta}} = 657{,}000 \text{ in lbf sec}^2 \tag{349}$$

$$\text{Total rudder inertia} = \begin{array}{l} 1.12\text{x}10^5 \\ \underline{6.57\text{x}10^5} \\ 7.69\text{x}10^5 \text{ in lbf sec}^2 \end{array} \tag{350}$$

APPENDIX II

BALL BEARING FATIGUE LIFE CRITERIA

In two of the systems proposed, the problem arose of predicting the expected lifetime of anti-friction bearings. That is, the system had to be designed such that the expected lifetime was satisfactory. Ball bearing fatigue life can be estimated with reasonable accuracy and in general decreases in proportion to the cube of the load. In most ball bearing applications the lifetime may be estimated by considering only the maximum load condition. This is because the force is to the cube power which quickly reduces the lifetime contributions from lower loadings to negligible values. The steering engine on the other hand operates such a small proportion of the time at maximum load that its lifetime calculation depends primarily on its operating conditions under light loadings. In order to calculate this lifetime properly a load history diagram is needed which plots hours of operation during a given period versus percent load. The data for such a curve would have to be gathered from records of underway operations in existing ships and is not presently available. The curve on this design would then have to be extrapolated to the expected lifetime of the ship popularly estimated as twenty years. The weighted load life formula may now be used. For the case of variable loading, constant speed, variable percentage operating time, this formula is:[34]

P_{wt} = weighted average load

$$= P_{max}\left[T_1\left(\frac{P_1}{P_m}\right)^{3.33} + T_2\left(\frac{P_2}{P_m}\right)^{3.33} +\!+\!+\; T_n\left(\frac{P_n}{P_{max}}\right)^{3.33}\right]^{0.3} \qquad (351)$$

APPENDIX II

BALL BEARING FATIGUE LIFE FORMULA

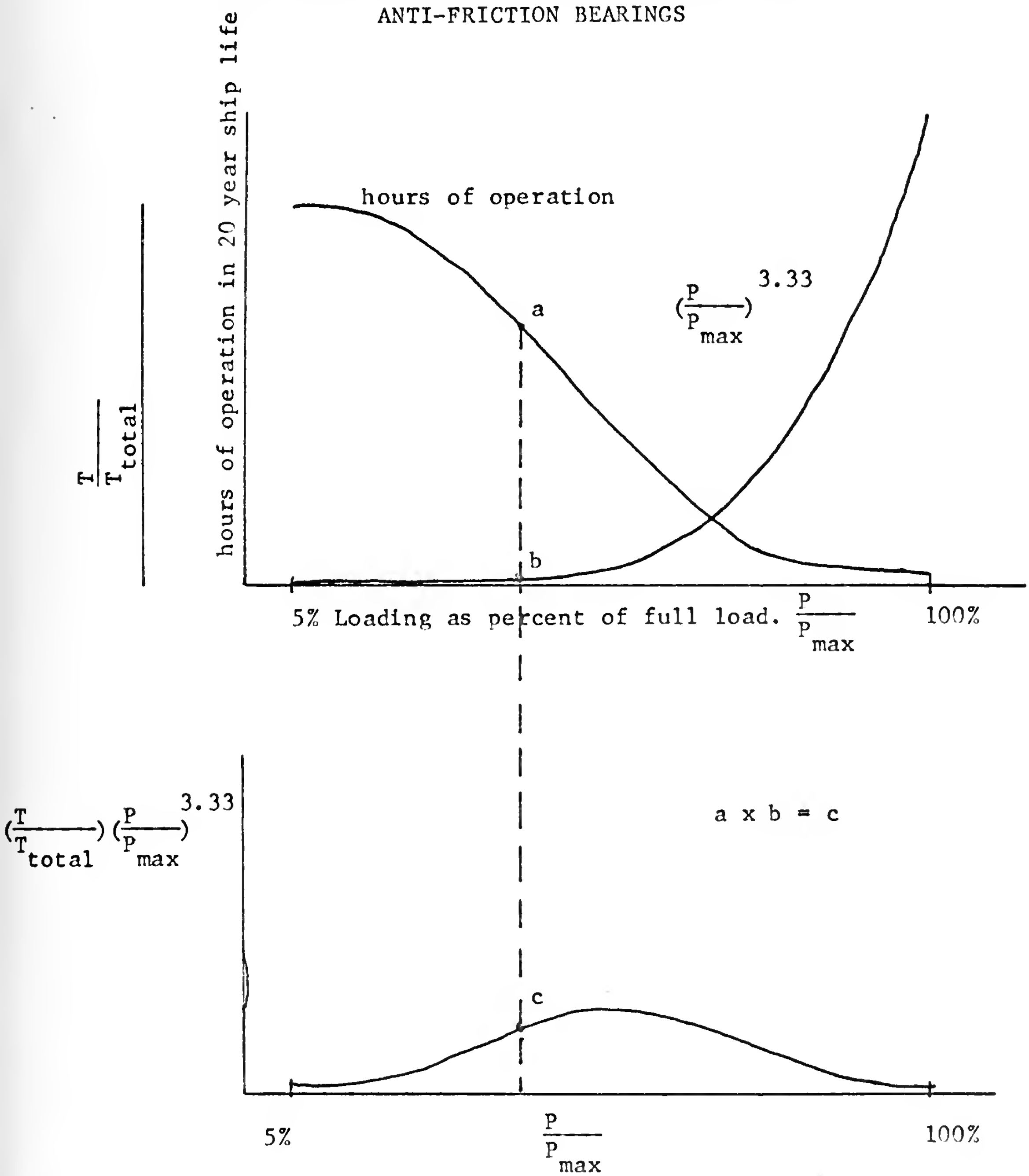
In two of the systems proposed, the problem [illegible] of [illegible] the expected lifetime of anti-friction bearings. [illegible] to be designed such that the expected lifetime [illegible] bearing fatigue life can be estimated with reasonable [illegible] in general decreases in proportion to the cube of the load. [illegible] bearing applications the lifetime may be estimated [illegible] the maximum load condition. This is because the [illegible] cube power which quickly reduces the lifetime contribution [illegible] lower loadings to negligible values. The steering engine [illegible] operates such a small proportion of the time at maximum load [illegible] lifetime calculation depends primarily on its operating conditions [illegible] loadings. In order to calculate this lifetime properly a load [illegible] diagram is needed which plots hours of operation during a given period versus percent load. The data for such a curve would have to be gathered from records of underway operations [illegible] is not presently available. The curve on this design would then have to be extrapolated to the expected lifetime of the ship [illegible] as twenty years. The [illegible] load life formula [illegible] for the case of variable loading, constant speed, variable percentage operating time, this formula is:[24]

P_m = weighted average load

[illegible]

FIGURE XXX

CALCULATION OF WEIGHTED AVERAGE LOADING FOR ANTI-FRICTION BEARINGS



where T_1, etc., are the fractions of the total operating time that the load P_1, etc., is imposed.

One method of calculating the rating is to use the total life history curve constructed from actual ship data. Integration of the area under this curve (probably most conveniently done using Simpson's Rule) will give the total operating lifetime required at the weighted mean load to be calculated. Using this total lifetime, recalibrate the vertical scale to read in percentage of total operating time. Now draw a curve for $(\frac{P}{P_{max}})^{3.33}$ on the same graph. By multiplying corresponding points of these two curves, a third curve may be constructed. This curve is now integrated, again by Simpson's Rule. The value of this integral raised to the .3 power gives the mean weighted load. That is the bearing must be capable of operating at this load for the total operating lifetime calculated above.

In the absence of the data to perform the above calculation and faced with the necessity of producing a reasonable estimate of lifetime requirements for anti-friction bearings used in steering systems, the Bureau of Ships has evolved the following calculation[34].

Assume that during the expected 20 year life of the ship that the rudder operates at maximum load 4000 times for an average duration of two minutes at each occurrance. Total time at maximum load is thus 8000 minutes. Next assume that two thirds load occurs a million times for average durations of one minute each for a total of 1×10^6 minutes. Finally assume one third loading occurs two million times for average durations of one minute each for a total of 2×10^6 minutes. Total time of operation is then 3,000,000 min or 50,000 hours.

where T_1, etc., are the fractions of the total operating time that the load P_1, etc., is imposed.

One method of calculating the rating is to [illegible] a total life history curve constructed from actual [illegible]. The [illegible] of the area under this curve (probably most conveniently done using Simpson's Rule) will give the total operating lifetime and permit the weighted mean load to be calculated. Using this total lifetime [illegible] the vertical scale to read in percentage of total operating time, now draw a curve for $\left(\frac{P}{P_{max}}\right)^{3.33}$ on the same graph. By multiplying corresponding points of these two curves, a third curve may be constructed. This curve is now integrated, again by Simpson's Rule. The [illegible] of the integral raised to the .3 power gives the [illegible] load. [illegible] the bearing must be capable of operating at this load for the total operating lifetime calculated above.

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The weighted mean load is now calculated.

$$P_{wt} = P_{max}\ [.00267(1)^{3.33} + 3.32(.667)^{3.33} + 666(.333)^{3.33}]^{.3}$$

$$= P_{max}\ [.00267 + .0864 + .0173]^{.3} = P_{max}\ [.10637]^{.3} \qquad (352)$$

$$P_{wt} = .51\ P_{max}$$

Thus the requirement is that the bearings be capable of operating for 50,000 hours at 51% of maximum load. It is apparent that this is a conservative estimate intended to indicate what a reasonable value for the lifetime should be. In the absence of more accurate data its use will probably result in satisfactory bearing design.

APPENDIX III

CALCULATION OF HYDRAULIC PISTON AND CYLINDER CURVES

TABLE I

CALCULATION OF DATA POINTS FOR FIGURE VI

p	$\frac{f\sigma_y}{p}$	$(\frac{f\sigma_y}{p})$	(K^2-1)	$\frac{f\sigma_y}{p}(K^2-1)$
$.01f\sigma_y$	100	10^4	.0203	2.03
$.05f\sigma_y$	20	4×10^2	.108	2.16
$.1f\sigma_y$	10	100	.236	2.36
$.2f\sigma_y$	5	25	.586	2.93
$.3f\sigma_y$	3.33	11.1	1.16	3.86
$.5f\sigma_y$	2	4	4.14	8.28

$\frac{f\sigma_y}{p}[\frac{2t_e}{\ell}(1-\frac{\gamma_o}{\gamma_s})+\frac{\gamma_o}{\gamma_s}]$				$\frac{f\sigma_y}{p}(K^2-1)+\frac{f\sigma_y}{p}[\frac{2t_e}{\ell}(1-\frac{\gamma_o}{\gamma_s})+\frac{\gamma_o}{\gamma_s}]$			
$\frac{t_e}{\ell}=.2$	$\frac{t_e}{\ell}=.1$	$\frac{t_e}{\ell}=.5$	$\frac{t_e}{\ell}=.01$	$\frac{t_e}{\ell}=.2$	$\frac{t_e}{\ell}=.1$	$\frac{t_e}{\ell}=.05$	$\frac{t_e}{\ell}=.01$
46.1	28	19	11.8	48.13	30.03	17.03	12.8
9.22	5.6	3.8	2.36	11.38	7.76	5.96	4.52
4.61	2.8	1.9	1.18	6.97	5.16	4.26	3.54
2.305	1.40	.95	.59	5.235	4.33	3.88	3.52
3.86	1.54	.93	.63	5.40	4.79	4.49	4.253
8.28	.92	.56	.38	9.20	8.84	8.66	8.516

$$\frac{\gamma_o}{\gamma_s} = \frac{.8}{7.87} = .101$$

TABLE II

CALCULATION OF DATA POINTS FOR FIGURE IX

	$\frac{f\sigma_y}{p}\left[1.27\sqrt{\frac{T}{f\sigma_y R^3}} + .101\right]$				Total Expression			
p	$\frac{T}{f\sigma_y R^3} = .001$	$\frac{T}{f\sigma_y R^3} = .01$	$\frac{T}{f\sigma_y R^3} = .05$	$\frac{T}{f\sigma_y R^3} = .1$	.001	.01	.05	.1
$.01f\sigma_y$	14.11	22.8	38.5	47.1	16.14	24,83	40.53	49.13
$.05f\sigma_y$	2.82	4.56	7.7	9.41	4.98	6.72	9.86	11.51
$.1f\sigma_y$	1.411	2.28	3.85	4.71	3.771	4.46	6.21	7.07
$.2f\sigma_y$	.706	1.14	1.925	2.35	3.636	4.07	4.855	5.28
$.3f\sigma_y$	.47	.76	1.282	1.57	4.33	4.62	5.142	5.43
$.5f\sigma_y$	.282	.456	.77	.941	8.562	8.736	9.05	9.221

$$2\left(1 - \frac{\gamma_o}{\gamma_s}\right)\sqrt{\frac{(\cos^2 35° + .2 \sin 35° \cos 35°)(\alpha)}{\tan^2 35°}} + \frac{\gamma_o}{\gamma_s} = (2)(.9)(.706) + .101 = 1.27 + .101$$

take $\alpha = 1$ (conservative)

APPENDIX IV

DERIVATION OF EQUATIONS FOR TILLER AND LINKAGE, RAPSON SLIDE, AND RACK AND PINION

Consider the equations of operation of the tiller and linkage, the rapson slide and the rack and pinion.

Tiller and Linkage

See Figure VIII for notation.

$$T = RF = RF_{\ell} \cos(\theta + \phi) \tag{353}$$

$$F_{\ell} = F_p \cos\phi \tag{354}$$

Now in order to write the equation in terms of θ, θ_m, and ℓ only, write the equation for ϕ.

$$d_m = R - R \cos \theta_m = R(1-\cos \theta_m) \tag{355}$$

$$\sin\phi = \frac{d}{\ell} = \frac{d_m - R(1-\cos\theta)}{\ell} \tag{356}$$

$$\sin\phi = \frac{R[(1-\cos\theta_m)-(1-\cos\theta)]}{\ell} = \frac{R}{\ell}(\cos\theta-\cos\theta_m) \tag{357}$$

$$T = RF_p \cos\phi\cos(\theta+\phi) = FR_p \cos\phi(\cos\theta\cos\phi - \sin\theta\sin\phi) \tag{358}$$

$$= RF_p[\cos^2\phi\cos\theta - \sin\theta\cos\phi\sin\phi] = RF_p[\cos^2\phi\cos\theta - \frac{\sin\theta \sin 2\phi}{2}] \tag{359}$$

$$T = RF_p[(1-\sin^2\phi)\cos\theta - \sin\theta\sin\phi\sqrt{1-\sin^2\phi}] \tag{360}$$

$$= RF_p[(1- \frac{r^2}{\ell^2}(\cos\theta - \cos\theta_m)^{]})\cos\theta - \sin\theta \frac{r}{\ell}(\cos\theta-\cos\theta_m)\sqrt{1 - \frac{r^2}{\ell^2}(\cos\theta-\cos\theta_m)^2}] \tag{361}$$

but

$$[\cos\theta - \cos\theta_m]^2 = [-2\sin(\frac{\theta+\theta_m}{2})\sin(\frac{\theta-\theta_m}{2})]^2$$

$$= 4\sin^2(\frac{\theta+\theta_m}{2})\sin^2(\frac{\theta_m-\theta}{2}) \tag{362}$$

$$F_p = \frac{T}{R}\ \frac{1}{\cos\phi\cos(\theta+\phi)} \tag{363}$$

or in terms of θ only

$$F_p = \frac{T}{R}\ \frac{1}{[(1-(\frac{r}{\ell})^2[\cos\theta-\cos\theta_m]^2)\cos\theta - \sin\theta\,\frac{r}{\ell}(\cos\theta-\cos\theta_m)\sqrt{1-(\frac{r}{\ell})^2(\cos\theta-\cos\theta_m)^2}]} \tag{364}$$

<u>Rapson Slide</u>

$$T = rF = r\,\frac{F_p}{\cos\theta}$$

but $\quad \frac{R}{r} = \cos\theta \qquad (365)$

then $\quad T = \frac{RF_p}{\cos^2\theta} \quad$ and $F_p = \frac{T}{R}\cos^2\theta \qquad (366)$

$$F_R = \frac{T}{r}\sin\theta \tag{367}$$

travel $\quad = 2R\tan\theta_{max} \qquad (368)$

<u>Analysis of Friction Forces in Rapson Slide</u>

$$F_p = F\cos\theta + F_{F_1} + F_{F_2}\sin\theta \tag{369}$$

but $\quad F_{F_1} = fF_R$ and $F_{f_2} = fF \quad$ where f is the coefficient of sliding friction

$$= fF\sin\theta \tag{370}$$

$$F_p = F\cos\theta + fF\sin\theta + fF\sin\theta$$

$$= F(\cos\theta + 2f\sin\theta) \tag{371}$$

then if use $f = .1$ from Kents handbook

$$F_p = F(\cos \quad + .2 \sin \ )$$

$$= \frac{T}{R} (\cos^2\theta + .2 \sin\theta\cos\theta) \tag{372}$$

Rack and Pinion

$$T = RF_p \quad \text{or} \quad F_p = \frac{T}{R} \tag{373}$$

$$F_r = \frac{T}{R} \tan \psi \tag{374}$$

where ψ is the gear tooth pressure angle.

Compare the travels of the tiller and linkage, the rapson slide, and the rack and pinion for the same maximum rudder torque, the same required maximum piston force F_p and the same maximum rudder angle $\theta_{max} = 35°$.

Tiller and Linkage

$$F_p = \frac{T}{R} \frac{1}{\cos\phi\cos(\theta+\phi)} \quad \text{where } \phi=0 \text{ when } \theta = 35° = \theta_m \tag{375}$$

$$= \frac{T}{R} \frac{1}{(1)\cos 35°} \tag{376}$$

$$R = \frac{T}{F_p \cos 35°} \tag{377}$$

$$\text{total travel} = 2(R \sin 35°) = \frac{2T}{F_p} \frac{\sin 35°}{\cos 35°} = \frac{2T}{F_p} \tan 35° = \frac{2T}{F_p} (.7) \tag{378}$$

Rapson Slide

$$F_p = \frac{T}{r} \cos \theta_m \tag{379}$$

$$r = \frac{T}{F_p} \cos 35° \tag{380}$$

$$\text{total travel} = 2 r \sin 35° = \frac{2T}{F_p} \cos 35° \sin 35° = \frac{2T}{F_p} (.47) \tag{381}$$

Rack and Pinion

$$r = \frac{T}{F_p} \tag{382}$$

$$\text{total travel} = \frac{70}{180}\pi 2r = \frac{2T}{F_p}\pi\frac{7}{18} = \frac{2T}{F_p}1.221 \tag{383}$$

If θ_{max} is increased to 45°, the total travel for the three mechanisms is:

$$\text{Tiller and linkage} = \frac{2T}{F_p}(1) \tag{384}$$

$$\text{Rapson slide} = \frac{2T}{F_p}(.48) \quad \text{use } \theta_{max} = 37.45° \tag{385}$$

$$\text{Rack and pinion} = \frac{2T}{F_p}(1.57) \tag{386}$$

On the basis of the total travels just computed, some conclusions can be drawn about the merits of these three mechanisms when used in conjunction with a hydraulic piston and cylinder. Since F_p is the same in each case, the area of the hydraulic piston required to drive each one is the same. The working length of the hydraulic cylinder must be equal to the travel. Therefore in comparing travels, the lengths and hence the weights of equal diameter hydraulic cylinders are compared. It is apparent from the above calculations that the rapson slide has the shortest travel, and therefore its use will result in the lightest piston and cylinder. The reasons for this can be seen quickly by looking at the rapson slide equations. Travel is directly proportional to the nominal radius R. Because this device increases its effective lever arm so quickly with θ, the required torque can be developed with a very small nominal radius R. This in turn results in the low travel. Although

the rapson slide will result in the lightest piston and cylinder, it is important to note that at this point it is not clear that it will result in the lightest overall system.

APPENDIX V

ESTIMATION OF WEIGHTS FOR RAPSON SLIDES

Hub Calculation

Torsional shear in the hub.

$$\tau = \frac{2T.r_o}{\pi(r_o^4-r_i^4)} = \frac{2T\ kr_i}{r_i^4(k^4-1)} = \frac{2T}{\pi r_i^3}\left(\frac{k}{k^4-1}\right) = .227\ \sigma_y \quad (387)$$

$k = \frac{r_o}{r_i}$ r_o = outer radius, r_i = inner radius

$\tau_y = .65\ \sigma_y\ \sigma_{max} = .35\ \sigma_y$

$$\tau_{max} = (.65)(.35)\sigma_y = .227\ \sigma_y \quad (388)$$

let $r_i = 12''$, $r_o = 13$

$$\text{the} \quad = \frac{(2)(5 \times 10^6)(13)}{\pi(13^4 - 12^4)} = \frac{130 \times 10^6}{\pi .79\times 10^4} = 52.4 \times 10^2 = 5240 \text{ psi} \quad (389)$$

Try 2 keys 2" deep

$$\text{Load on 1 key} = \frac{5 \times 10^6}{(2)\ (12)} = 2.08 \times 10^5 \text{ lbs} \quad (390)$$

If σ_y = 50,000, Max direct shear = $(.6)(.35)\sigma_y = 2.10\ \sigma_y$

$$= 10{,}500 \text{ psi} \quad (391)$$

Shear stress in key $= \frac{2.08 \times 10^5}{2\chi} = 10{,}500$ where = length of key

$$\chi = 9.92 \text{ in} \quad (392)$$

Let length be 12"

$$\text{Bearing pressure} = \frac{2.08 \times 10^5}{1 \times 12} = 17{,}300 \quad (393)$$

Maximum allowable compression force = (1.60)(.35)(50,000)=28,000

$$W\ \text{hub} = \gamma_s{}^2\pi(r_o^2-r_i^2)x = \gamma_s 2\pi r_i^2(k^2 - 1)x \quad (394)$$

The hub weight is really dependent on keying practice used because the stress limit is shear across the key. Since hub thickness in a constant because it must accommodate the key, hub length depends on key shear. The number of keys cannot be increased because of the problem of load distribution.

Rapson Slides for Various Values of R

(The calculations are based on a value of $f\sigma_y$ = 17,500 psi.)

R = 19"

Minimum R for a 24" diameter stock is determined as follows.

rudder stock radius	12"
hub thickness	2"
1/2 bearing pad	5"
	19" = R

(395)

$$r = \frac{19}{\cos 35°} = 23.2", \quad F = \frac{5 \times 10^6}{23.2} = 216{,}000 \text{ lbs} = 108{,}000/\text{beam} \quad (396)$$

@ 2000 psi, bearing area = 108 in^2 = 10" x 10.8"

$$\text{let } h = 5.5", \text{ pin dia} = 10", \ \beta = 12" \ d = \frac{28-12}{2} = 8 \quad (397)$$

$$\text{beam length } \ell = (r + 1/2 \text{ bearing pad} + 1/4') - \frac{D}{2} + \alpha = 15.715" \quad (398)$$

$$\alpha = 14\left(1-\sqrt{1-\frac{9}{49}}\right) = 14(1-.903) = 1.265", \ b = 10.465" \ a = 5.25"$$

$$\text{Shear at end} = \frac{(108000)(10.465^2)}{2(15.715)^3}(5.25 + (2)(15.715)) = 57{,}100 \text{ psi}$$

$$\text{Required total web thickness} \quad t = \frac{57{,}100}{(8)(17500)} = .408" \quad (399)$$

$$\text{Required end plate thickness} \quad t_e = \frac{57{,}100}{(5.5)(17500)} = .593" \quad (400)$$

Bending moment = (57,100)(15.715) = 897000 ft lbs

$$\text{Required I} = \frac{(897000)(4)}{17500} = 205 \text{ in}^4$$

$$\text{I of web} = \frac{(.408)(8^3)}{12} = \frac{17.4}{187.4}$$

$$= R = \text{Required I of flanges}$$

Required flange thickness,

$$t_f(h-t)(2\ (\tfrac{d}{2})^2 = (5.5-4.1)(2(4)^2) = 187.4$$

$$t_f = 1.15 \qquad (401)$$

Crosshead pin

Shear area $= \frac{108000}{17500} = 6.17\ in^2$ required

Bending moment $= (108,000)(5.5) = .595 \times 10^6$ in lb.

$\sigma_{max} = \frac{My}{I} = 17500 = \frac{(.595 \times 10^6)(5)}{\frac{(r_o^4 - r_i^4)}{2}}$ where y = distance from center of gravity of cross section to outer fiber

$$625 - r_i^4 = \frac{(.59 \times 10^6)(5)(2)}{(\pi)(17500)} = 108.2$$

$$r_i = (625 - 10.82)^{1/4} = 516.8^{1/4} = 4.76 \qquad (402)$$

Area $= (r_o^2 - r_i^2) = (25-22.7) = 7.2\ in^2$

For bearing rigidity etc., let thickness be 1/2" instead of the 1/4" allowable.

Rods, $F_p = \frac{T}{R}\ \psi\ \frac{5 \times 10^6}{19}\ (.765) = 201,000$

Area $= \frac{201,000}{17500} = 11.5\ in^2$ (403)

Length = travel + (clearances - yoke thickness)

= 2 R tan 37° + 5.25 sin 37° + 14 sin 37° - 2"

= 28.6 + 3.16 + 8.42 - 2 length = 38.18" or 38 1/4" (404)

Crossbar

$$F_R = \frac{T}{r} \sin 35° = \frac{5 \times 10^6}{23.2} \sin 35° = 124000 \text{ lbs.}$$

Length = travel + 2 clearances

$28.6 + 2(3.16 + 8.42) = 51.76 \quad a = 40.18$

$b = 11.58$

Bending moment $M = \dfrac{F_R \, a^2 b}{2}$

$$= \frac{(124000)(40.18)^2(11.58)}{(51.76)^2} = 866{,}000. \text{in lbs}$$

$$\text{Shear} = \frac{Pa^2}{3}(a + 3b) = \frac{(124000)(40.18^2)}{51.76^9}[40.18 + 3(11.58)] = 108{,}000 \text{ lbs.}$$

Use bearing face 8" wide by 1/2" thick stiffened transversely.

Required separation of flanges

$$(1/2)(8)(2(\frac{x^2}{2})) = \frac{866{,}000 \; \frac{x}{2}}{17{,}500}$$

$$x = \frac{866{,}000}{(4)(17500)} = 12" \qquad (405)$$

$$\text{Required web thickness} = \frac{108{,}000}{(12)(17500)} = .515" \qquad (406)$$

Weight = web	(.286)(.515)(11)(63.76)	=	103
flanges	2(.286)(.5)(8)(63.76)	=	146
stiffener allowance 10%			25
			275 lbs (407)

<u>Weights</u>

$$\text{Hub} = \gamma_s(r_o^2 - r_i^2)(12) = (286)(\pi)(14^2 - 12^2)(12) \qquad = 562 \text{ lbs}$$

$$\text{Web} = 4\,\gamma_s\, t d \ell = (4)(.286)(.408)(8)(15.715) \qquad = 59 \text{ lbs}$$

$$\text{Flanges} = 8\gamma_s\, t_f(h-t)\ell = (8)(.286)(1.15)(5.09)(15.715) \qquad = 210 \text{ lbs}$$

$$\text{End plate} = 2\gamma_s t_e h D = (2)(.286)(.593)(5.5)(28) \qquad - 52 \text{ lbs}$$

Allowance for fillets and stiffeners 10% beams = 25 lbs

tiller weight = 908 lbs

Bearing blocks $= 2\gamma_s(5.5)[(10)(12) - \pi 5^2](2.86)(11)(120-78.5)$ $= 131$ lbs

Crosshead pin $\gamma_s \pi(r_o^2 - r_i^2)4h = (2.86)(\pi)(25-20.2)(22)$ $= 95$ lbs

Yoke approximation same as bearing blocks $= 131$ lbs

crosshead weight 357 lbs

Rods $= 2\gamma_s A\ell = 2(.286)(11.5)(38.25)$ $= 252$ lbs

Crossbar $= 274$ lbs

Summary	tiller	908
	crosshead	357
	rods	252
	crossbar	274
	total	1791 lbs

(408)

R = 25"

$$r = \frac{25}{\cos 35°} = 30.5 \quad F = \frac{5 \times 10^6}{30.5} = 164000 \text{ lbs} = 82000/\text{beam} \tag{409}$$

Bearing area $= 82$ in^2 $= 9" \times 9.1"$, $h = 4.5"$

$$\text{Pin dia.} = 9", \ B = 11", \ d = \frac{28-11}{2} = 8.5" \tag{410}$$

$$\ell_1 = r-R + 1/4" + \text{bearing} = 30.5-25 + .25 + 9 = 14.75" \tag{411}$$

$a = 4.75"$

$b = 10.0"$

$$\text{Shear} = \frac{(82000)(10^2)}{2(14.75^3)}[4.75-(2)(14.75)] \qquad 44{,}200 \text{ psi}$$

$$t = \frac{442000}{(8.5)(17500)} = .297" \tag{412}$$

$$\text{Required end plate thickness} = \frac{44200}{(4.5)(17500)} = .561" \tag{413}$$

$$\text{Required } I = \frac{(44{,}200)(14.75)(4.25)}{17500} = 159$$

$$\text{I of web} = \frac{(.297)(8.5^3)}{12} = \frac{15.2}{143.8}$$

Required flange thickness $t_f = \dfrac{143.8}{(4.5-.297)(2(3.75)^2)} = 1.22''$ (414)

Install crossplate of thickness 1.22 and h = 4.5

Crosshead pin = let thickness = 1/2", r_o = 9", height = 18" (415)

Bending moment = (82000)(4.5) = $.369 \times 10^6$ in lbs

$$\sigma_{max} = \frac{(.369 \times 10^6)(4.5)}{\left(\frac{4.5^4 - 4^4}{2}\right)} = \frac{(.369 \times 10^6)(9)}{(\pi)(156)} = 6{,}780 \text{ psi}$$

$$\sigma_s = \frac{82000}{\pi(4.5^2-4^2)} = \frac{82000}{\pi(4.2)} = 6{,}540 \text{ psi}$$

Rods, $F_p = \dfrac{5 \times 10^6}{25} \ \psi$ = 153000 lbs

Area = $\dfrac{153000}{17500}$ = 8.75 in^2 (416)

Length = (2)(25)(tan 37°) + 4.75 sin 37° + 14 sin 37°-2

= 37.7 + 2.71 + 8.42-2 = 45.03" (417)

<u>Crossbar</u>

$$F_R = \frac{5 \times 10^6}{30.5} \sin 35° = 94{,}000$$

Length = 37.7 + 2(2.71 + 8.42) = 59.96"

a = 48.83"

b = 11.13"

$$M = \frac{(94000)(48.83^2)(11.13)}{59.62^2} = 694{,}000 \text{ in lbs}$$

$$\text{Shear} = \frac{(94000)(48.83)^2}{(59.96)^3} [48.83 + 3(11.13)] = 85{,}600 \text{ psi}$$

Bearing area = 47 = 7" x 7"

Use bearing face 7" x 1/2" thick

$$\text{Required separation } x = \frac{694{,}000}{(3.5)(17500)} = 11'' \tag{418}$$

$$\text{Required web thickness} = \frac{85600}{(11)(17500)} = .445'' \tag{419}$$

Weight = web (.286)(.445)(10)(72)	=	92
flange 2(.286)(.5)(7)(72)	=	144
stiffener allowance	=	25
		261

<u>Weights</u>

Hub	562
Flanges 8(.286)(1.22)(4.203)(14.75)	173
Webs 4(.286)(.297)(8.5)(14.75)	43
Root section flanges 4(.286)(1.22)(4.203)(6.5)	38
web 2(.286)(.297)(28)(6.5)	31
Cross plate 2(.286)(1.22)(28)(4.203)	83
End plate 2(.286)(.561)(4.5)(28)	41
Stiffener and fillet allowance	35
tiller	1006
Bearing blocks = $2(.286)(4.5)[(9)(11) - \pi 4.5^2]$	91
Crosshead pin = .286 (20.2-16)(18)	68
Yoke	91
crosshead	250
Rods = 2(.286)(8.75)(45.03)	226
Crossbar	261

Summary	tiller	1006
	crosshead	250
	rods	226
	crossbar	261
	total	1743

(420)

<u>R = 30"</u>

$$r = \frac{30}{\cos 35°} = 36.6", \; F = \frac{5 \times 10^6}{36.6} = 136{,}500 \text{ lbs} = 68{,}250/\text{beam} \quad (421)$$

Bearing area = 68.3 = 8" x 8.5", h = 4.25"

$$\text{Pin dia} = 8", \; \beta = 10", \; d = \frac{28-10}{2} = 9" \quad (422)$$

$$\ell_1 = r + \frac{\text{bearing}}{2} + 1/4" - (R - \frac{\text{bearing}}{2}) = r-R + 1/4" + \text{bearing}$$

$$= 36.6-30 + .25 + 8 = 14.85"$$

$$a = 4.25" \quad (423)$$

$$b = 10.60"$$

$$\text{Shear} = \frac{(68{,}250)(10.60^2)}{2(14{,}85)^3}[4.25 + 2(14.85)] = 39{,}800 \text{ psi}$$

$$t = \frac{39{,}800}{(9)(17500)} = .253" \quad (424)$$

$$\text{Required end plate thickness} = \frac{39800}{(4.25)(17500)} = .535" \quad (425)$$

$$\text{Required } I = \frac{(39{,}800)(14.85)(4.5)}{17500} = 152$$

$$I \text{ of web} = \frac{(.253)(9^3)}{12} \qquad \underline{15.3}$$

$$137.3$$

Required flange thickness

$$t_f = \frac{137.3}{(4.25-.253)(2(4.00)^2)} = 1.08" \quad (426)$$

Install cross plate of thickness 1.08" and h = 4.25".

Now check to see if this flange and web are sufficient in the root section

Inertia = I web	280
I flanges = (4.25-.29)(1.08)(365)	1390
	1670

$M = (68,250)(36.6 - 14)$

$= 1,540,000$ in lbs

$\sigma = \frac{(1,540,000)(14)}{1670} = 12,900$ psi checks O.K.

Crosshead pin $t = 1/2"$ $r_o = 4.0"$ $h = 17"$

Bending and shear are O.K.

Rods $F_p = \frac{5 \times 10^6}{30} \psi = 127,500$ lbs

$$\text{Area} = \frac{127,500}{17500} = 7.3 \text{ in}^2 \tag{427}$$

Length = (2)(30)(tan 37°)+ 4.25 sin 37°+ 14 sin 37°-2

$$45.2 + 2.44 + 8.42 - 2 = 54.06 \tag{428}$$

<u>Crossbar</u>

$F_R = \frac{5 \times 10^6}{36.6} \sin 35° = 78,400$ psi

$$\text{Length} = 45.2 + 2(2.44 + 8.42) = 66.92" \tag{429}$$

$a = 56.06"$

$b = 10.86$

$M = \frac{(78,000)(56.06)(10.86)}{(66.92^2)}(56.06 + 3(10.86)) = 72,700$ psi

Bearing area = 39.2 = 6" x 6.5"

Use bearing face 6" x 1/2" thick

$$\text{Required separation } x = \frac{569,000}{(3)(17500)} = 11" \tag{430}$$

$$\text{Required web thickness} = \frac{72700}{(11)(17500)} = .378'' \tag{431}$$

Weight = web (.286)(.378)(10)(70)		= 86
flanges 2(.286)(.5)(6)(79)		= 136
stiffener allowance		= 20
		242 lbs

Weights

Hub		562
Flanges 8(.286)(1.08)(4.00)(14.85)		147
Webs 4(.286)(.253)(9)(14.85)		39
Root section flanges 4(.286)(1.08)(4.00)(12)		60
webs 2(.286)(.253)(28)(12)		50
Crossplate 2(.286)(1.08)(4.00)(28)		70
End plate 2(.286)(5.35)(4.25)(28)		37
Allowance for stiffeners and fillets		35
	tiller	1000
Bearing blocks = $2(.286)[(8)(10)-\pi 4^2](4.25)$		73
Crosshead pin = $.286\ \pi(16-12.2)(17)$		58
Yoke		73
	crosshead	204
Rods = 2(.286)(7.3)(54.1)		226
Crossbar		242

Summary

tiller	1000
crosshead	204
rods	226
crossbar	242
total	1672 lbs

(432)

R = 40"

$$r = \frac{40}{\cos 35°} = 48.8", \quad F = \frac{5 \times 10^6}{48.8} = 102{,}500 \text{ lbs} = 51{,}250/\text{beam} \quad (433)$$

Bearing area = 5.2 x 7" x 7.5"

$$\text{Pin dia.} = 7", \ \beta = 9", \ d = \frac{28-9}{2} = 9.5" \ h = 3.75" \quad (434)$$

$$\ell_1 = 48.8 - 40 + .25 + 7 = 16.05" \qquad a = 3.75" \quad (435)$$

$$b = 12.30"$$

$$\text{Shear} = \frac{(51{,}250)(12.30^2)}{2(16.05)^3}[3.75 + 2(16.05)] = 33{,}600 \text{ psi}$$

$$t = \frac{33600}{(9.5)(17500)} = .203" \quad (436)$$

$$\text{Required thickness} = \frac{33600}{(3.75)(17500)} = .511" \quad (437)$$

$$\text{Required I} = \frac{(33600)(16.05)(4.75)}{17500} = 147$$

$$\text{I of web} = \frac{(.203)(9.5^3)}{12} = \frac{14.5}{132.5}$$

$$\text{Required flange thickness } t_f = \frac{132.5}{(3.75-.203)(2(4.25^2))} = 1.04" \quad (438)$$

Install crossplate of thickness 1.04" and h of 3.75".

Check at root

Inertia I web = 370

I flanges = (1.04)(3.75-.203)(365) = 1347

1717

$$M = (51{,}250)(48.8 - 14) = 1{,}785{,}000 \text{ in lbs}$$

$$= \frac{(178500)(14)}{1717} = 14{,}500 \text{ psi}$$

Crosshead pin, t = 1/2" r_o = 3.5" h_p = 15" stresses O.K.

Check bending $\sigma = \dfrac{(51250)(5.625)(3.5)(2)}{\pi(3.5^4 - 3^4)} = 9{,}300$ psi

Rods, $F_p = \frac{T}{R}\psi = \dfrac{5 \times 10^6}{40} .765 = 95{,}500$ lbs

Area $= \dfrac{95{,}500}{17{,}500} = 5.45$ in^2 (439)

Length = (2)(40)(tan 37°) + (3.75)(sin 37°) + 14 sin 37° − 2

60.3 + 2.26 + 8.42 − 2 = 68.98 = 69" (440)

Check buckling $P_{crit} = \dfrac{EA^2}{2} = \dfrac{(\ )(30 \times 10^6)(4.8)^2}{69^2} = 450{,}000$ O.K.

Crossbar

$F_R = \dfrac{5 \times 10^6}{48.8} \sin 35° = 58{,}700$ lbs

Length 60.3 + 2 (2.26 + 8.42) = 81.66" (441)

a = 70.98"

b = 10.68"

$M = \dfrac{(58700)(70.98^2)(10.68)}{81.66^2} = 475{,}000$

Shear $= \dfrac{(58{,}700)(70.98^2)}{81.66^3}\,[70.98 + 3(10.68)] = 56{,}200$ psi

Bearing area = 29.4 = 5" x 6"

Use bearing face 5" x 1/2" thick

Required separation $x = \dfrac{475{,}000}{(2.5)(17{,}500)} = 11''$ (442)

Required web thickness $= \dfrac{56200}{(11)(17500)} = .293''$ (443)

Weight = web (.286)(.293)(10)(94)	79
flanges 2(.286)(.5)(5)(94)	135
stiffener allowance	20
	234 lbs

Weights

Hub		562
Flanges $8(.286)(1.04)(3.547)(16.05)$		136
Webs $4(.286)(.203)(9.5)(16.05)$		36
Root section flanges $4(.286)(1.04)(3.547)(22.5)$		95
webs $2(.286)(.203)(28)(22.5)$		73
Cross plate $2(.286)(1.04)(3.547)(28)$		60
End plate $2(.286)(.511)(3.75)(28)$		31
Allowance for stiffeners and fillets		35
	tiller	1028
Bearing blocks $= 2(.286)[(9)(7)-\pi 3.5^2](3.75)$		53
crosshead pin $= .286\ \pi(3.5^2-3^2)15$		44
Yoke		53
	crosshead	150
Rods $= 2(.286)(5.45)(69)$		215
Crossbar		234

tiller	1028	
crosshead	150	
rods	215	
crossbar	234	
	1627	(444)

R = 50"

$$r = \frac{50}{\cos 35°} = 61.1'', \quad F = 81{,}900 \text{ lbs} = 40{,}950/\text{beam} \tag{445}$$

Bearing area 40.95 = 6" x 6.75"

$$\text{Pin dia } 6.75'', \quad \beta = 8.75'', \quad d = \frac{28-8.75}{2} = 9.625'' \quad h = 3'' \tag{446}$$

$\ell_1 = 61.1-50 + .25 + 6.25 = 18.10''$ $\quad a = 3.625''$ (447)

$b = 14.375''$

$$\text{Shear} = \frac{(40{,}950)(14.375^2)}{2(18.10)^3}\,[3.625 + 2(18.10)] = 28.400 \text{ psi}$$

$$t = \frac{28400}{(9.625)(17500)} = .169'' \qquad (448)$$

$$\text{Required end thickness} = \frac{28400}{(3)(17500)} = .541'' \qquad (449)$$

$$\text{Required I} = \frac{(28400)(18.10)(4.812)}{17500} = 141$$

$$\text{I of web} = \frac{(.169)(9.625^3)}{12} = \underline{12.6}$$

$$128.4$$

$$\text{Required flange thickness } t_f = \frac{128.4}{(3-.169)(2(4.812^2))} = .978'' \qquad (450)$$

Install cross plate of thickness .978" and h of 3"

Check at root

$$\text{Inertia I web} = \frac{(.169)(28^3)}{12} = 309$$

$$\text{I flanges} = .978(3-.169)(365) = \underline{1010}$$

$$1319$$

$$M = (40950)(61.1-14) = 1{,}930{,}000 \text{ in lbs}$$

$$= \frac{(1{,}930{,}000)(14)}{1319} = 22{,}500 \quad \text{Not acceptable}$$

Required I = 1550

$$\text{I web} = \underline{309}$$

$$1241$$

$$\text{Required flange thickness } t_f = \frac{1241}{(3-.169)(2(14^2))} = 1.12'' \qquad (451)$$

Crosshead pin $t = 1/2''$, $r_o = 3.375''$, $h_p = 13.5''$

Check bending $\sigma = \dfrac{(40950)(4.5)(3.375)(2)}{(3.375^4 - 2.875^4)} = 6,440$ O.K.

Rods $F_p = \dfrac{T}{R}\,\psi = \dfrac{5 \times 10^6}{50}\,.765 = 76,500$ lbs

$$\text{Area} = \frac{76500}{17500} = 4.38 \text{ in}^2 \tag{452}$$

Length $= (2)(50)(\tan 37°) + 3.625 \sin 37° + 14 \sin 37° - 2$

$$75.3 + 2.18 + 8.42 - 2 = 83.90'' \tag{453}$$

Check buckling $P_{crit} = \dfrac{30 \times 10^6\ 4.38^2}{83.92} = 257,000$ lbs O.K.

Crossbar

$$F_R = \frac{5 \times 10^6}{61.1} \sin 35° = 47,000$$

$$\text{Length} = 75.3 + 2(2.18 + 8.42) = 96.5'' \tag{454}$$

$$a = 85.9''$$

$$b = 10.6''$$

$$M = \frac{(47000)(85.9^2)(10.6)}{96.5^2} = 394,000$$

Shear $= \dfrac{(47000)(85.9^2)}{96.5^2}\,[85.9 + 3(10.6)] = 44,400$

Bearing area = 23.5 = 5" x 5"

Use bearing face 5" x 1/2" thick

$$\text{Required separation } x = \frac{394000}{(2.5)(17500)} = 9'' \tag{455}$$

$$\text{Required web thickness} = \frac{44400}{(9)(17500)} = .282'' \tag{456}$$

Weight = web	(.286)(.293)(8)(108)	73
flanges	2(.286)(.5)(5)(108)	155

stiffener allowance	23
	251

Weights

Hub		562
Flanges 8(.286)(1.12)(2.831)(18.1)		131
Webs 4(.286)(.169)(9.625)(18.1)		34
Root section flanges 4(.286)(1.12)(2.831)(32.725)		119
webs 2(.286)(.169)(28)(32.725)		89
Crossplates 2(.286)(1.12)(2.83)(28)		51
End plates 2(.286)(.541)(3)(28)		26
Allowance for stiffeners and fillets		45
	tiller	1057
Bearing blocks $= 2(.286)(8.75)(6.75-\pi \frac{6.75^2}{4})(3)$		40
Crosshead pin $= (.286)(\pi)(3.375^2-2.875^2)(13.5)$		39
Yoke		40
		119
Rods 2(.286)(4.38)(83.9)		210
Crossbar		251

Summary

Tiller	1057	
Crosshead	119	
Rods	210	
Crossbar	251	
Total	1637 lbs	(457)

Rack and Pinion Calculation

Obtain Min dia from K factor calculation

$$K = 100 = \frac{W}{F_d} \left(\frac{m_g + 1}{m_g}\right) \tag{458}$$

$$m_g = 1$$

$$W_t = \frac{2T_p}{d}$$

$$100 = \frac{2T_p}{F_d^2} \left(\frac{2}{1}\right)$$

if limit face width to 1.5 d, can solve for d

$$d^3 = \frac{(2)(5 \times 10^6)}{(1.5)(100)} = 66,700$$

$$d = 40'' \tag{459}$$

$$F = 60''$$

Minimum R = 20

Find maximum number of teeth based on strength calculation.

$$S_{at} = 17500$$

$$S_t = \frac{W_t K_o}{K_V} \quad \frac{P}{F} \frac{K_s K_m}{J}$$

$J = .36$-18 teeth $\qquad P = \frac{N}{d}$

$.4$-25 teeth $\qquad F = 1.5\ d$

$$W_t = \frac{2T}{d}$$

$$K_m = 2.0$$

$$K_s = 1.0$$

$$K_o = 1.25$$

$$K_V = 1.0$$

$$17500 = \frac{(2)(5 \times 10^6)(1.25)}{d\ 1.0} \quad \frac{N}{1.5\ d^2} \quad \frac{(1.0)(2.0)}{.4}$$

$$N = \frac{(17500)(40^3)(.4)(1.5)}{(10 \times 10^6)(1.25)(2.0)} = 26.9 \tag{460}$$

Separating force $= \frac{T}{R} \tan \phi$

$$= \frac{5 \times 10^6}{20} \tan 20°$$

$$= 91,000 \text{ lbs} \tag{461}$$

@ 2000 psi bearing area = 45.5 in^2

Without performing any calculation it can be seen that a gear five feet high and forty inches in diameter will be too heavy. The $f\sigma_y$ chosen is probably the cause of these stupendous proportions as much higher values are normally used in gear design. However, this is the same yield used in calculating the weights of the other systems. If high yield steels were used, and if the required torques were lower more competive dimensions would result.

Tiller and Linkage

The tiller and linkage can be analysed by comparing it to the rapson slide components. The tiller arm would be lighter because it could be of a more efficient shape. In place of the cross bar and rods there is the linkage arm. It is difficult to tell much about it since selection of its length is arbitrary. However, it must carry a much heavier load than the rapson slide and hence it must have a reasonably heavy section. The linkage may be a little lighter than the slide but it is much more cumbersome. Further more it is doubtful if the possible weight advantage here offsets the added piston and cylinder weight.

Calculation of Piston and Cylinder Weights

For first calculation select a steel with tensile yield strength of 50,000 psi. Then if $f = .35$, $f\sigma_y = 17500$ psi.

$R = 19$

$$\frac{T}{f\sigma_y R^3} = \frac{5 \times 10^6}{(17500)(19^3)} = .042$$

From graph Figure IX optimum

$$\frac{p}{f\sigma_y} = .21 \quad P_{opt} = 3{,}670 \qquad (462)$$

$C_W = 4.75$ where C_W is the nondimensional weight

$$W = \frac{(4.75)(2)(5 \times 10^6)(.765)(.7)(.283)}{17500} = \quad 411 \text{ lbs}$$

For two cylinders	822 lbs
20% allowance for foundations etc.	164 lbs
Total	986 lbs (463)

$R = 25$

$$\frac{T}{f\sigma_y R^3} = \frac{5 \times 10^6}{(17500)(25^3)} = .0183$$

From graph optimum

$$\frac{p}{f\sigma_y} = .187, \quad P_{opt} = 3{,}270 \text{ psi} \qquad (464)$$

$$C_W = 4.16$$

$$W = \frac{(4.16)(1515000)}{17500} = \quad 360 \text{ lbs}$$

times	2	=	720 lbs
times	.20	=	144 lbs
total			864 lbs (465)

$R = 30$

$$\frac{T}{f\sigma_y R^3} = \frac{5 \times 10^6}{(17500)(30^3)} \quad .0106$$

From graph optimum

$$\frac{p}{f\sigma_y} = .184, \; P_{opt} = 3{,}220 \text{ psi} \tag{466}$$

$$C_W = 4.04$$

$$W = \frac{(4.04)(1515000)}{17500} = 350 \text{ lbs}$$

times 2 = 700 lbs

times .20 = <u>140 lbs</u>

Total 840 lbs (467)

$R = 40$

$$\frac{T}{f\sigma_y R^3} = \frac{5 \times 10^6}{(17500)(40^3)} = .00446$$

From graph optimum

$$\frac{p}{f\sigma_y} = .172, \; P_{opt} = 3{,}010 \text{ psi} \tag{468}$$

$$C_W = 3.80$$

$$W = \frac{(3.80)(1515000)}{17500} = 329 \text{ lbs}$$

times 2 = 658 lbs

times .20 = <u>132 lbs</u>

total 790 lbs (469)

$R = 50$

$$\frac{T}{f\sigma_y R^3} = \frac{5 \times 10^6}{(17500)(50^3)} = .00229$$

From graph optimum

$$\frac{p}{f\sigma_y} = .170, \; P_{opt} = 2,980 \text{ psi} \tag{470}$$

$$C_W = 3.7$$

$$W = \frac{(3.7)(1515000)}{17500} \qquad 320 \text{ lbs}$$

times	2	=	640 lbs
times	.20	=	128 lbs
total			768 lbs (471)

For second calculation choose a high strength steel with a tensile strength of 100,000 psi. Then $f\sigma_y$ = 35000 psi. By doubling $f\sigma_y$ it can be seen that the optimum pressures have been doubled. However they now exceed the maximum practical limit of 5000 psi. The method of calculation that must be used now is to enter the graph with the value of $p/f\sigma_y$ for 5000 psi and at the appropriate cross curve pick off weight.

For 5000 psi, $\frac{p}{f\sigma_y} = \frac{5000}{35000} = .143$

$$R = 19 \; C_W = 4.77, \; W = \frac{(4.77)(1515000)}{35000} = 207$$

times	2	=	414
times	.20	=	83
total			497 lbs (472)

$$R = 25 \; C_W = 4.47, \; W = \frac{(4.47)(1515000)}{35000} = 194$$

times	2	=	388
times	.20	=	78
total			466 lbs (473)

$R = 30\ C_W = 4.20,\ W = \frac{(4.20)(1515000)}{35000} = 182$

times 2 = 364

times .20 = 73

total 437 lbs (474)

$R = 40\ C_W = 3.8,\ W = \frac{(3.8)(1515000)}{35000} = 165$

times 2 = 330

times .20 = 66

total 396 lbs (475)

$R = 50\ C_W = 3.7,\ W = \frac{(3.7)(1515000)}{35000} = 160$

times 2 = 320

times .20 = 64

total 384 lbs (476)

Calculation of Pump Capacity and Drive Motor Horsepower

$R = 19\ F_p = 201{,}000,\ A = 54.8$, travel 26.6

flow rate $\frac{(54.8)(26.6)}{30} = 48.6\ \text{in}^3/\text{sec} \times 4.329 \times 10^{-3}\ \frac{\text{gal}}{\text{in}^3}$

$\times \frac{60\ \text{sec}}{\text{min}} = 12.6$ gpm

Requires 33.2 H.P.[15] Assume a 5% friction loss in the cylinder = 35 HP (477)

$R = 25,\ F_p = 153{,}000,\ A = 46.8$, travel 35

flow rate = 54.6 = 14.2 gpm, 30.0 HP = 34.8 HP (478)

$R = 30,\ F_p = 127{,}500,\ A = 39.6$, travel 42

flow rate = 55.5 = 14.4 gpm, 32.8 HP, 34.5 HP (479)

$R = 40,\ F_p = 95{,}500,\ A = 3.17$, travel 56

flow rate = 59.2 = 15.4 gpm, 32.1 HP, 33.8 HP (480)

$R = 50$, $F_p = 76{,}500$ lbs, $A = 25.7$, travel 70

flow rate = 60 = 15.6 gpm requires 32.0 HP, 33.7 HP (481)

Weights may now be determined

25 HP A.C. 3 phase motor (overload to 37.5 HP)	=	360 lbs
Hydraulic pump (Hydreco Model 45)		110 lbs
Oil reservoir and piping		200 lbs
		670 lbs (482)

APPENDIX VI

CALCULATION OF CONSTANTS FOR HYDRAULIC TRANSMISSION

$Q_{ideal} = \frac{na\dot{\theta}_p}{2} x = k_p x$ where x is the angular displacement of the motor control shaft. [19]

$$(483)$$

$Q_n = Q_i - Q_L - Q_C$ = volume delivered to motor (484)

$Q_L = K_L P$ where K_L = total pump and motor leakage (485)

$$Q_n = D_m \dot{\theta}_m \qquad (486)$$

Q_C = may be neglected. Although pressure is very high there is very little fluid present in the system.

$$D_m \dot{\theta}_m = K_p x - K_L P \qquad (487)$$

but $PQ_n = T_m \dot{\theta}_m$

$$P = T_m \frac{\theta_m}{Q_n} = \frac{T_m}{D_m} \qquad (488)$$

then

$$D_m \dot{\theta}_m = K_p x - \frac{K_L}{D_m} T = C_p x - \frac{C_L}{D_m} T \qquad (489)$$

Constants for Pump

$Q_i = 33.6$ gpm at $x_{max} = 14°$

$$K_p = \frac{33.6}{x_{max}} = \frac{(33.6)(231)}{(14)(60)} = 9 \frac{in^3}{\text{sec degree}} = 515 \frac{in^3}{\text{sec radian}} \qquad (490)$$

$Q_i = 33.6 \frac{x}{x_{max}} = 515\ x$ where x is in radians

$Q_L = 33.6 - 32.4$ gal @ 4000 psi

$= 1.2$

$$K_{LP} = \frac{1.2}{4000} = .0003 \times \frac{231}{60} = .001155 \frac{in^5}{16\ sec} \qquad (491)$$

$$C_L = .00385 + .001155 = .005005 \tag{492}$$

$$36.8 \text{ gpm} = D_m \; 1750 \text{ rpm}$$

$$D_m = \frac{36.8}{1750}$$

$$D_m = .0021 \frac{\text{gal}}{\text{rev}} = .772 \frac{\text{in}^3}{\text{rad}} \tag{493}$$

<u>Constants for Motor</u> (constant volume)[15]

$$Q_L = 40.8 - 36.8 = 4 \text{ gpm @ } 4000 \text{ psi}$$

$$= K_{L_m} P$$

$$K_{LM} = \frac{4}{4000} = .001 \frac{\text{gal in}^2}{\text{lbf min}} \times \frac{231 \text{ in}^3}{\text{gal}} \times \frac{\text{min}}{60 \text{ sec}} = .00385 \frac{\text{in}^5}{\text{lb sec}} \tag{494}$$

K_{LM} could more accurately be represented as a function of motor speed at 0 speed, $Q_L = 2$ gpm

$$Q_L = (2 \text{ gpm} + \frac{N}{1750} \times 2 \text{ gpm}) = 2 \text{ gpm } (1 + \frac{N}{1750}) \text{ where N=motor rpm}$$

$$K_{LM} = .0005 \; (1 + \frac{N}{1750}) \frac{\text{gal in}^2}{\text{min lbf}} \tag{495}$$

<u>Damping Calculation</u>

60 HP in, 163 ft lbs out @ 1500 rpm

pump = 32.8 gpm @ 2760 psi

$$\text{pump leakage } (-)\begin{array}{r} 33.6 \\ \underline{32.8} \\ .8 \end{array} \text{gpm} \qquad \text{motor leakage } (-)\begin{array}{r} 34 \\ \underline{31.5} \\ 2.5 \end{array} \text{gpm}$$

$$\text{leakage HP} = \frac{3.3 \text{ gal}}{\text{min}} \; \frac{231 \text{ in}^3}{\text{gal}} \; \frac{2760 \text{ lbs}}{\text{in}^2} \; \frac{\text{ft}}{12 \text{ in}} \; \frac{3.3\text{x}10^{-5} \text{HP}}{\text{ft lbs/min}} = 5.31 \text{ HP} \tag{496}$$

$$\text{theoretical HP } \frac{1500 \text{ rev}}{\text{min}} \; \frac{2 \text{ rad}}{\text{rev}} \; 163 \text{ lb ft } 3.03\text{x}10^{-5} \; \frac{\text{HP}}{\text{ft lbs/min}}$$

$$= 46.6 \text{ HP} \tag{497}$$

Damping HP = 60 HP - 5.31 HP - 46.6 HP = 8.09 HP (498)

$$\text{Damping torque} - 8.09\ \text{HP}\ \frac{12}{3.03\text{x}10^{-5}2\pi 1500} = 339\ \text{in lbs}$$

Damping constant

$$339\ \text{in lbs x } 1/\left[\frac{1500\ \text{rev}}{\text{min}}\ \frac{2\pi\text{rad}}{\text{rev}}\ \frac{\text{min}}{60\ \text{sec}}\right] = .216\ \text{in lb sec} = B_m \tag{499}$$

Est. motor inertia 5" dia 5" long

$$J_m = 1.15 \times 10^{-3}\left[\left(\frac{5}{2}\right)^4\right](5) = .225\ \text{in lb sec}^2 \tag{500}$$